Jacobs J. W.

An experiment in the teaching of plane geometry comparing the formal method with the informal method.

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AN EXPERIMENT IN THE TEACHING OF PLANE GEOMETRY

COMPARING

THE FORMAL METHOD WITH THE INFORMAL METHOD

Submitted by

John William Jacobs

(B.S. in E.E., Northeastern University, 1932)

(B.S. in Ed., Bridgewater State Teachers College, 1937)

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First Reader: Henry W. Syer, Assistant Professor of Education

Second Reader: Donald D. Durrell, Professor of Education

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Appendix B

# INTRODUCTION

Do students of plane geometry show a greater achievement as measured by standardized tests when taught by the formal method or when taught by the informal method?

For the purpose of this paper, the terms formal method and informal method will have to be carefully defined. When the idea for this paper was first presented to the seminar group, the title read as follows: An Experiment in Teaching Plane Geometry Using the Traditional Method versus the Postulational Method. The traditional method was defined as consisting of units of work where the formal proof of theorems was required before they could be used in proofs of original exercises. The postulational method was defined as assuming or postulating the proof of theorems required to prove the original exercises of the unit.

After much discussion, it was decided that less confusion would result if the terms traditional method and postulational method were changed to formal method and informal method. Formal method will be defined as the method of teaching plane geometry by requiring the learning or committing to memory of formal proofs of theorems for reproduction before attempting to use them in

the solution of original exercises. In the informal method the proofs of certain basic theorems which are essential to the proof of other theorems and the nature of their proof is such that an artificial method such as superposition or indirect proof is required will be postulated. Understanding of the theorems will be gained by intuitive, informative, experimental, or inductive teaching. The use of all of the terms is practically synonymous, and the authorities in the field recommend that the word "informal" be used.

Comparison of the results obtained by two groups of plane geometry students of approximately equal ability taught by the two methods defined in the previous paragraphs should tend to influence the type of course to be given and perhaps the textbook to be used.

This study will be set up as an experiment with two groups equated as to mental maturity by the California Short-Form Test of Mental Maturity.1/

The two groups were tested halfway through the course and again at the end of the course. The gain or loss between half-year achievement and the final achievement for each group were compared to see if any significant difference could be determined.

1/Elizabeth T. Sullivan, Willis W. Clark, and Ernest W. Tiegs, California Short-Form Test of Mental Maturity, Advanced S-Form, (California Test Bureau, Los Angeles, 1939)

## CHAPTER I

## PREVIOUS INVESTIGATION IN THE FIELD

There appear to be no readily available studies which attempt to prove experimentally that a theorem a day is not the way to teach plane geometry.

In arriving at this conclusion, the writer first examined the card catalogue files of theses in the Boston University School of Education Library. This examination revealed how few theses in the field of mathematics and particularly in plane geometry were available. It was my good fortune, however, to discover that my seminar adviser, Professor Henry W. Syer, had recently compiled an extensive list of theses in the field of mathematics which are to be found in the libraries of schools and colleges all over the United States. This list covered a period of about twenty years. Working over this list, I compiled a sub-list of thirty theses pertaining to plane geometry. About half of this group were found to be studies comparing the directed study or laboratory method of teaching with the traditional lesson-learning plan. Of the remaining fifteen, six dealt with motivating materials in plane geometry. It was not possible to tell from the titles in every instance what the remaining studies actually were reporting on.

Through the courtesy of the Boston University School of Education Library and the Inter-University Library Loan Service, several of the theses with the doubtful titles and several which were thought might apply were requested for examination. In every instance, however, the theses examined were found not to have any direct bearing on the work this paper is experimenting with.

The studies examined brought to light some experimental procedures, however, in the handling of conclusions which the writer thought might apply to this study.

There are numerous indications in recent geometry textbooks that much thought has been given to the subject of postulation. In addition to the opinion of textbook writers, many articles by teachers of mathematics were found which indicate that the subject has been discussed pro and con. In the investigation of the literature of the field which follows, an attempt will be made to indicate what textbook writers are doing. Second, some of the arguments both for and against postulation will be given which were located in professional periodicals and books.

It has been interesting to note how the postulation idea has developed from the idea of postulating only the congruence theorems as expressed by Shibli

to that of postulating theorems on congruence, parallelism, similarity, and inequalities as indicated by Reeve, Birkhoff and Beatley, and others.

The quotations which follow were chosen for three main reasons: First, they discussed what should be postulated; second, why postulate; third, they recorded opposition to the idea. The first is taken care of by the articles which follow by Reeve, Shibli, Christofferson, and Welkowitz. The second by the quotes from Mensenkamp, Fawcett, Purdue Workshop, and Nygaard. The third, the recorded opposition, as evidenced by articles by Royall, Salkind, Barber, and Lynch.

Herberg and Orleans1/ in their A New Geometry for Secondary Schools state "By treating as assumptions the conditions for congruence, parallelism, and similarity, a thoroughly logical course has been built that is considerably simpler than the usual course."

In a similar vein, Crawford and Schnell2/ in Clear Thinking, An Approach through Plane Geometry offer as one of their twelve basic principles, "No. 2, Formal geometry progresses more rapidly and with more understanding when

1/ Theodore Herberg and Joseph B. Orleans, A New Geometry for Secondary Schools, (Boston; D.C.Heath and Company, 1948), Preface p. 3

2/ Leroy H. Schnell and Mildred Crawford, Clear Thinking, An Approach through Plane Geometry, (New York; Harper & Brothers, 1943), Preface p. 10

extreme care is taken in developing fundamental concepts. No good or lasting purpose is served by rushing into a study of formal demonstrative proofs."

Birkhoff and Beatley[1] in Basic Geometry. "The traditional approach to demonstrative geometry involves careful study of certain theorems which the beginner is eager to accept without proof and which he might properly be led to take for granted as assumption or postulate. Such an approach obscures at the very outset the meaning of 'proof' and 'demonstration'. The employment of superposition in the proof of some of these theorems is even more demoralizing. This method of proof is so out of harmony with the larger aims of geometry instruction that despite the validity, its use is commonly restricted to those few cases for which no better method can be found."

Reichgott and Spiller[2] in Today's Geometry. "Formal demonstration is kept at a minimum. No attempt is made to adhere to a rigorous proof. There are more postulates, assumptions, and assumed theorems than in traditional geometry. Numerous exercises based on these theorems and postulates afford practice in logical development."

1/ George D. Birkhoff and Ralph Beatley, Basic Geometry, (Scott, Foresman and Company, 1941, Chicago), Preface p. 3

2/ David Reichgott and Lee R. Spiller, Today's Geometry, (New York, Prentice-Hall, Inc., 1938), Preface p. 7

Let us next consider what teachers in the field have to say on the subject. Reeve[1] in The Teaching of Geometry gives the following list of particularly important basic theorems:

1. The congruence theorems.
2. The equality of alternate angles in case of parallels.
3. The sum of the angles of a triangle.
4. The theorems on similar figures.
5. The Pythagorean Theorem.
6. The measurement of angle between two chords.

He suggests that we might well postulate all but the third and could do most originals. "So important are these few propositions that if we had no others with which to work we could with these alone prove a large proportion of the original exercises of plane geometry." and he continues, "Proof of propositions in textbooks should be models or should not be proved at all. In fact we could easily omit the proofs of many conventional propositions and the pupils would gain in every respect by such omission. Eg. Proof for each of the family of parallelogram propositions usually begins with 'Diagonal of a parallelogram divides

1/ National Council of Teachers of Mathematics, The Teaching of Geometry, Fifth Yearbook of the National Council of Teachers of Mathematics, (New York; Bureau of Publications of Teachers College, Columbia University,) p 1-28

it into two congruent triangles.' This and all such simple propositions should be treated as original exercises."

Birkhoff and Beatley[1/] in A New Approach to Elementary Geometry present the following argument, "What is the point in telling beginners that we shall assume certain 'self-evident truths', and then asking them to prove certain other propositions which they regard as equally self-evident truths? Would they not come to a greater understanding of the nature of a proof through the effort to prove easy 'originals' which are not too plausible and which seem therefore to require justification?"

Mensenkamp[2/] in Some Desirable Characteristics in a Modern Plane Geometry Text presents these factors to be considered: "First, there is the pupil himself. The number enrolled constitutes a much larger proportion of the total population than it did a decade or so ago. This means there are now present in our tenth grade classes many more students representing the lower levels of mental ability than was formerly the case. Most students of this type do not intend to go to college and it is hard to interest them in a difficult subject like

1/ National Council of Teachers of Mathematics, The Teaching of Geometry, Fifth Yearbook of the National Council of Teachers of Mathematics, (New York; Bureau of Publications of Teachers College, Columbia University), p 86-96

2/ Ibid., p 199-206

geometry, especially if the textbook presentation is of an abstract or formal character. The two reports which have given direction and sanction to geometry reform in this country during the past several years are: The reorganization of mathematics in secondary schools, a report by the National Committee on Mathematics Requirements (1923), and a report of the College Entrance Examination Board on Geometry requirements."

Fawcett[1] in The Nature of Proof states "Actual classroom practice indicates that major emphasis is placed on a body of theorems to be learned rather than on the method by which these theorems are established. Pupil feels theorems are important in themselves and in his earnest effort to know them resorts to memorization."

Shibli[2] in Recent Developments in the Teaching of Plane Geometry. "Some teachers advocate postulating congruence by sides along with the other two congruence theorems in the interests of simplicity and consistency. Some fear that the movement toward free postulation may go too far."

1/ National Council of Teachers of Mathematics, The Nature of Proof, Thirteenth Yearbook of National Council of Teachers of Mathematics (New York; Bureau of publications of Teachers College, Columbia University), p. 117

2/ J. Shibli, Recent Developments in the Teaching of Plane Geometry, (1932, Penn State College, published by J. Shibli), p. 104

Christofferson[1] in Geometry Professionalized for Teachers suggests the postulation of congruent triangles rather than proof by superposition. "The chief defense for complete postulation of all three theorems, in addition to the abandonment of superposition is simplicity and understanding at the beginning of the course."

He goes on to say that the number of Fundamental Theorems chosen should be based upon whether they are to be used again in the proof of other theorems and mentions the possibility of using only ten constructions and twenty theorems or a total of thirty as compared to the average of 195 constructions, theorems, and corollaries in six often-used textbooks. See Appendix A, Table 1 for table showing this comparison taken from Christofferson's study.

It is interesting to note that the purpose of this study was to discover how few really fundamental theorems are needed upon which to build the entire structure of geometry. This was from a professional and not a mathematical point of view.

1/ H.C.Christofferson, Geometry Professionalized for Teachers, (George Banta Publishing Company, Menasha, Wisconsin, 1933), p. 34-37

The following extracts from the list of "suggested places in plane geometry in which we can shorten, eliminate, or postulate to allow time to introduce space concepts" were taken from the report of the committee[1] working on the first six weeks in plane geometry of the second annual Purdue Mathematics Workshop. These extracts emphasize a point this paper is trying to determine -- whether or not too much time is being given to formal proof.

1. It is suggested that we postulate the theorem "one and only one line can be drawn from (at) a given point perpendicular to a given line."

2. If the hypotenuse - angle congruency proposition is proved by superposition, it might be postulated.

3. It might be possible to postulate the theorem "If two angles have their sides respectively perpendicular, they are either equal or supplementary."

4. The theorem "The sum of the angles of a polygon of n sides is (n-2) straight angles" may be postulated or proved informally.

5. In connection with inequalities in a triangle or a circle, certain theorems and corollaries might be postulated.

6. The concurrency propositions, having been intuitively established in the introduction, may be postulated.

7. Postulate the theorem "If two polygons are similar, they can be divided into triangles which are similar and similarly placed" and its converse.

8. Postulate the continuity of the Pythagorean Theorem as applied to similar polygons constructed upon the three sides of a right triangle.

A complete list will be found in Appendix A, Table 2.

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1/ Committee on the First Six Weeks in Plane Geometry, Second Annual Purdue Mathematics Workshop, June 16-June 28, 1947

Following along the same line, Nygaard[1] in A Functional Revision of Plane Geometry says, "The writer has made some effort to determine what material, usually included in plane geometry, is of little or no value in later mathematics or science courses. He is convinced that a number of the theorems dealing with the circle have no future use -- for instance, the measurement of all sorts of angles in terms of their intercepted arcs." "Theorems based on dividing a line segment externally are in the same class." "Comprehensive proofs of the theorems dealing with the area of rectangles, parallelograms, triangles, and trapezoids would come under the same ban." "Many of the relationships involved in triangles, parallelograms, and circles could be more efficiently presented as lists of characteristic properties or as student exercises than as theorems completely proved in the textbook."

Readers who are interested may wish to read A Reply to Mr. Nygaard[2] by Norman N. Royall, Jr., Winthrop College, Rock Hill, South Carolina, or a further

1/ P.H. Nygaard, A Functional Revision of Plane Geometry, The Mathematics Teacher (October 1941), Vol. 34, No. 6, pp 269-273

2/ Norman N. Royall, Jr., A Reply to Mr. Nygaard, The Mathematics Teacher (April 1942), Vol. 35, No. 4, pp 179-181

discussion of Mr. Nygaard's article entitled The War on Euclid, by Charles Salkind [1].

Royall is concerned with the harm that may be done to sound instruction in mathematics by the type of discussion quoted above. He suggests that when Mr. Nygaard or anyone else makes "A Functional Revision of Plane Geometry" one cannot be sure that what is left is plane geometry, and he points out that one of the primary objectives of a plane geometry course is to give the students a chance to learn the nature of a deductive proof.

Salkind agrees with several of the premises, but feels that intuitive geometry is taken care of by our Junior High Schools. He says, "However, whether this type of geometry teaching, call it intuitive or informational or experimental or inductive, precedes the unit of demonstrative geometry or is taught simultaneously with it, it is imperative for us, as purveyors of Mathematical Knowledge, to know the nature of demonstrative geometry."

1/ Charles Salkind, The War on Euclid, The Mathematics Teacher (May 1942), Volume 35, No. 5, pp 205-207

Harry C. Barber[1] in Random Notes on Geometry Teaching says "There has been much discussion of what should be done with superposition proofs. The plan often advocated is to postulate all the theorems usually so proved. In England the reason given is that when we move a figure we cannot be certain that it does not change in size and shape. In the United States the reasons given are that superposition is not a suitable method at the outset; it is not readily understood by the beginner; and it interferes with the later use of other methods of proof." Barber goes on to point out that all direct measurement is a process of superposition. He says, "To omit the superposition proofs at the beginning of plane geometry is to miss the strongest link we have between everyday experience and the argument of geometry. Here is a place where teachers need to do battle against a current unfortunate trend."

Lynch[2] says, "outstanding among the many modes of attack that came rapidly to the fore, and one that apparently disposed of the difficulty satisfactorily, took its point of departure from one of Goethe's maxims: 'The greatest art in theoretical and practical life consists

1/ Harry C. Barber, Random Notes on Geometry Teaching, The Mathematics Teacher, (January 1938), Vol. 31, No. 1, p 31

2/ James M. Lynch, Individual Differences and Course Revision in Plane Geometry, The Mathematics Teacher, (March 1942), Vol. 35, No. 3, p 122

in changing a *problem* into a *postulate*.' Accordingly, following this procedure, it was urged that if the pupils' capacity for learning the congruence theorems, for example, is low, do not *try* to teach those theorems -- just postulate them, that is, treat them as axiomatic, as data self-evident immediately and requiring no proof. Is the class too dull to understand the proposition: if two straight lines are cut by a transversal, and the alternate interior angles are equal, the lines are parallel? Then postulate that proposition, too, and all annoyances and confusions about how to 'get it across' will vanish as though blown away on the wings of a gentle breeze."

Lynch[1/] continues in the same vein: "Merely turning the problems of teaching plane geometry to the 'masses' into postulates has the rather attractive advantage of being a very pleasantly painless procedure. It has the added advantage of increasingly gaining favor and support, as a teaching device, among professors of education, curriculum experts, and textbook authors. Indeed, one of the distinctive marks of a modern progressive text is the number of postulate assumptions and assumed theorems it contains -- the greater the amount of postulation, the more 'modern' and 'progressive' it is."

1/ James M. Lynch, *Individual Differences and Course Revision in Plane Geometry*, *The Mathematics Teacher*, (March 1942), Vol. 35, No. 3, p. 122

Lynch prefers to attack the problem by working on the mode of presentation as an important factor in understanding; and he suggests that by changing the style of presentation rather than the content of the course, the great mass of ordinary pupils will be able to cope with the material commonly reserved for a selected few.

Further investigation into suggested omissions and additions for the course in plane geometry, brings to light the following articles: Reeve[1] in A Proposal for Mathematics Education in the Secondary Schools of the United States says "In geometry it will be necessary to omit: (1) About two-thirds of the traditional propositions to be proved fully. The real purpose of logical geometry can better be secured by retaining only the necessary basal propositions, introducing more original matter, and reducing the deduction aspects of the course for many pupils." "In geometry we should add: (1) A modern beginning, establishing the truth of the propositions informally. This movement is already under way."

On choosing the theorems to keep, Samuel Welkowitz[2] has this to say, "The point to be stressed is not ground to

1/ William D. Reeve, A Proposal for Mathematics Education in the Secondary Schools of the United States, The Mathematics Teacher (January 1943), Vol. 36, No. 1, pp 11-20

2/ Samuel Welkowitz, Tenth Year Geometry for all American Youth, The Mathematics Teacher (March 1946), Vol. 39, No. 3, pp. 99-112

be covered but ground to be cultivated. More emphasis should be placed on the nature and meaning of deductive reasoning or the process of drawing necessary conclusions from a given set of assumptions and its application in all life thinking. Less emphasis should be placed on the solving of originals and more attention should be given to the appreciation of the nature of reasoning and types of reasoning.

"In line with the above the following guiding principles are recommended in selecting the propositions to be retained. Only those propositions should be retained for deductive proof or for factual knowledge or both which fulfill at least one of the following conditions: (1) They have many varied and interesting applications in the sciences, industry, shop, navigation and the arts of war (2) They form an indispensable link in the logical chain of reasoning. In the latter case it may sometimes be more desirable to assume the truth of the proposition on the basis of an informal proof or experiment."

I would like to close this discussion of the literature on the subject by reviewing some of the remarks of Rolland R. Smith[1/] on "How Much Formal Proof in Plane Geometry?" In this, he states, "We

1/ Rolland R. Smith, How Much Formal Proof in Plane Geometry? Subject of talk given at open conference on the teaching of mathematics in secondary schools at Boston University, School of Education, April 20, 1949

should have enough formal proof to fulfill our aims." In order to do this we should clarify our aims and he suggests that five or six large aims are certainly better than fifty or sixty small aims.

The changed type of College Entrance Board Examinations in mathematics has been responsible for some of the change in organization of material in plane geometry. Twenty years ago this examination would have consisted of six questions, two of them being book theorems to be proved. Since 1933, however, the examinations have been quite different. No proofs have been required for several years, but students have had to reason.

His point of view is best expressed by his statement of how to handle proof of propositions in classes of varying ability. He says, "Discuss the proof informally in all classes and reproduce it in the best classes."1/

1/ Rolland R. Smith, How Much Formal Proof in Plane Geometry? Subject of talk given at open conference on the teaching of mathematics in secondary schools at Boston University, School of Education, on April 20, 1949

## CHAPTER II

## EXPERIMENTAL PROCEDURES

The two groups to be experimented with consist of two classes each of eleventh-grade students enrolled in a traditional plane geometry course. One group, during the school year 1947-48, was taught by the formal method with the formal proof of theorems; e.g. the congruence of triangles by superposition learned or committed to memory. The other group, during the school year 1948-49, meeting at the same hours of the day, having the same teacher and textbook, used the informal method where the formal proof of the same theorems was omitted and the theorems were postulated for future use in solution of original examples.

These groups were taken from a junior-senior high school of about four hundred and fifty pupils total enrollment. A check with the school office showed a great deal of information to be available from the cumulative record cards as to previous grades, chronological age, ability, etc.

In addition this school has adopted the policy of using the Boston University School and College Relations Cooperative Testing Service which tests all pupils in the eighth grade with an educational battery of tests and again in the eleventh grade with a vocational battery of tests. Results of these tests are available for both groups

as eleventh graders, but for only the 1949 group as eighth graders. This would appear to be sufficient, however; and while some further refinement in grouping might have been possible with the eighth-grade data for both groups, this cannot be had for another year.

This data will be available in the following tables and charts in Appendix B:

1. Graph of Intelligence Quotient versus Frequency of All Eleventh-grade Students and of Plane Geometry Students in 1948.

2. Graph of Intelligence Quotient versus Frequency of All Eleventh-grade Students and of Plane Geometry Students in 1949.

3. Data Used in Calculating Mean Intelligence Quotients and Sigma of Unequated Groups.

4. Data Used in Calculating Mean Intelligence Quotients and Sigma of Equated Groups - First Trial.

5. Data Used in Calculating Mean Intelligence Quotients and Sigma of Equated Groups.

6. Intelligence Quotients and Previous Marks in Mathematics of 1948 Group.

7. Intelligence Quotients and Previous Marks in Mathematics of 1949 Group.

8. Data from Boston University School and College Relations Cooperative Testing Service Vocational Battery for Pupils in 1948 Group.

9. Data from Boston University School and College Relations Cooperative Testing Service Vocational Battery for Pupils in 1949 Group.

In setting up the experiment, it was proposed, first, to equate the groups by pairing of individuals. This was done by plotting intelligence quotients versus the average of mathematics grades for the 1948 and 1949 groups as shown in Chart 3, Appendix A. The results were not satisfactory, however, as a study of Chart 1 will show. First, in regard to intelligence quotients, note that there is only one horizontal pair line indicating the same ability level; and in twenty-four of the remaining thirty-four pairs, the 1948 group was below the 1949 group. Comparing the average of mathematics marks for grades seven, eight, nine, and ten, we find nine vertical pair lines indicating the same average grades; and in seventeen of the remaining twenty-six pairs, we find the 1948 group below the 1949 group.

Since the pairing of individuals produced only about thirty-five pairs of the forty odd pupils in each group compared, and a study of the pairs showed that the 1949 group had decided advantages, it was then decided to attempt to equate by groups.

Taking the data available for all students in each group, it was found that the mean intelligence quotient for the 1948 group was 105 with a sigma of 12.05, and for the 1949 group the intelligence quotient was 112

with a sigma of 11.98. This difference between the means was checked for significance by the formula $\text{Sigma}_D = \sqrt{\text{Sigma}_{M_{49}}{}^2 + \text{Sigma}_{M_{48}}{}^2}$ (see calculations Table 4, Appendix A). The standard error of the difference was found to be plus or minus 2.71.

Dividing the difference between the means by the standard error of the difference gave a ratio of 2.76, which from Table 34, page 213 in Garrett on Statistics[1] showed that the chances were 99.72 in 100 or almost a virtual certainty that the difference between the two groups was significant.

The above results necessitated the elimination of some of the lower members of the 1948 group and of the higher members in the 1949 group. The first trial of dropping the four lowest members of the 1948 group and the top two of the 1949 group gave a mean intelligence quotient of 106 for 1948 against 114 for 1949. On the second trial dropping the lowest nine for 1948 and the top four of 1949 gave a mean intelligence quotient of 108, sigma 10.2, for 1948 versus 110, sigma 10.1 for 1949. This difference between the means was then checked for significance by the same type of computation as the original difference (see Table 5, Appendix A), and the standard error of the

1/ Henry E. Garrett, Statistics in Psychology and Education, New York, Longmans, Green and Co., 1940, p. 213

difference was found to be plus or minus 2.43. Dividing the difference between the means by the standard error of the difference gave a ratio of .783, which from Table 34[1] entering at .80 showed that the chances were reduced to 79 in 100 that the true difference was greater than zero. Since the lower limit is negative, there is some chance that the true difference is less than zero. Inasmuch as the authors of the test give the probable error of estimate based on 600 pupils to be four points, this difference does not seem to be too great.

This now left thirty-five members in each group of plane geometry students with which to carry on the experiment.

These students have been equated upon the basis of intelligence quotient, but in line with the suggestion in Whitney[2] Elements of Research their equivalence has been checked for certain other significant characteristics summarized in the following table taken from the Data Tables in Appendix B.

1/ Henry E. Garrett, Statistics in Psychology and Education, New York, Longmans, Green and Co., 1940, p. 213

2/ Frederick L. Whitney, The Elements of Research, New York, Prentice-Hall, Inc., 1942, p. 225

Table I. Summary of Equivalence Factors

| Factor | 1948 Group Means | 1949 Group Means | Appendix B Tables |
|---|---|---|---|
| Intelligence Quotient | 108 | 110 | 5 |
| Chronological Age | 16 yrs 2 mos | 16 yrs 0 mos | 8 & 9 |
| Average of Previous Mathematics Marks | 2.43 - C+ | 2.47 - C+ | 6 & 7 |
| Problem Solving Ability | 7 | 7 | 8 & 9 |
| Reading Comprehension | 165 | 165 | 8 & 9 |
| Spatial Relations | 46 | 40 | 8 & 9 |

The next question that arose was what theorems should be postulated. The authorities in the field, Christofferson, Shibli, Herberg and Orleans, Birkhoff and Beatley, etc., are very much in agreement as to the main groups that should be postulated.

W.D.Reeve in the Fifth Yearbook1/ suggests the following be postulated: first, congruence of triangles; second, equality of alternate interior angles of parallel lines; third, theorems on similar figures; fourth, Pythagorean Theorem; fifth, measurement of angle between two chords.

1/ National Council of Teachers of Mathematics, The Teaching of Geometry, Fifth Yearbook of the National Council of Teachers of Mathematics, (New York; Bureau of Publications of Teachers College, Columbia University), p.23

For this experiment, it was decided that the group of theorems whose proofs depend upon artificial means such as superposition and indirect proof should be postulated. The second criteria applied was whether the theorems were needed in the proof of other theorems. Table 6, Appendix A gives the complete list of theorems postulated with the 1949 group. All other theorems were handled as original exercises. One interesting feature of the textbook used was that in the group of original exercises immediately preceding a theorem, in many instances, an exercise was used which was identical with the following theorem.

Of the textbooks examined (see Table 7, Appendix A) several were found which postulated or assumed the proof of practically all theorems. Several more were found which assumed or postulated the proof of the congruence theorem; but only one, Herberg and Orleans[1/] A New Geometry for Secondary Schools, followed the pattern outlined in the previous paragraph. The textbook used by the experimental group, Welchons & Krickenberger[2/] Plane Geometry, presented

1/ Theodore Herberg and Joseph B. Orleans, A New Geometry for Secondary Schools, (Boston, D.C.Heath and Co., 1948)

2/ A.M.Welchons and W.R.Krickenberger, Plane Geometry, (Boston, Ginn and Co., 1943)

a difficulty that could have been eliminated if the Herberg and Orleans text could have been used. Proofs of the assumed theorems were omitted from the body of the text but were given in a section at the end of the book.

This led to some discussion among the better students as to why the work was being omitted. In order not to bias this part of the experiment, the few who wanted to go into proofs such as superposition and having time to do so were told why these were being postulated and advised to go ahead on their own study of formal proofs and the instructor would be available for discussions outside of class.

There seem to be three main groups of textbooks with regard to this idea of postulating or assuming the truth of a theorem without formal proof: First, the group of which Schnell and Crawford[1/] Clear Thinking may be said to be typical where many assumed theorems are accepted and few formal proofs appear in the book; second, the group of which Herberg and Orleans[2/] A New Geometry for Secondary Schools is typical taking a middle ground

1/ Leroy H. Schnell and Mildred Crawford, Clear Thinking, An Approach through Plane Geometry, (New York, Harper & Brothers, 1943)

2/ Theodore Herberg and Joseph B. Orleans, A New Geometry for Secondary Schools, (Boston, D.C.Heath and Co., 1948)

between the formal and informal and while theorems are postulated, complete proofs are provided in the back of the book; third, the group of which Seymour and Smith[1] Plane Geometry is typical in which the congruence of triangles theorems and a few others are postulated. A list of geometry books falling in the various groups will be found in Table 7, Appendix A.

While the main factor on which the groups were equated was that of ability rating as determined by the California Test of Mental Maturity[2], Advanced form S, there were other factors which needed to be taken care of so that the one variable experiment could be carried on. Among these, the timetable of work for 1948 versus 1949 classes (see Table 8, Appendix A) was watched closely. The Teacher's Plan Book for 1948 was the source used for setting up this table. The work of the 1949 group was held as closely to that of the 1948 group as possible so that no undue advantage would be given either group. This was accomplished by allotting the same number of days to the 1949 group for each unit of work as was required by the 1948 group. The timetable of work for the 1948

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1/ F. Eugene Seymour and Paul J. Smith, Plane Geometry, Boston, Macmillan Co., 1941)

2/ Elizabeth T. Sullivan, Willis W. Clark, and Ernest W. Tiegs, California Short-Form Test of Mental Maturity, Advanced S-Form, (California Test Bureau, Los Angeles, 1939)

group versus the 1949 group gives the following information: the units in the order taught; number of days required by the 1948 group; number of days actually used by the 1949 group; a column showing the difference in time actually used by the two groups, a plus indicates less time required and a minus indicates more time required for the 1949 group as compared to the 1948 group.

It is interesting to note that due to the informal method used, a gain of about fifteen days' time was made during the first half year. Ten of these days were allotted to a unit on "space geometry" which had previously been left out of the course. The remaining five days were kept for possible use during the second half of the year's work. This time came in very handy as the group became so interested in the subject of loci constructions that an additional week was allotted to this unit of work.

The next problem to confront the writer was to determine whether the tests given to the 1948 group at the half year and at the end of the year could be used satisfactorily with the 1949 group for the desired result.

Since the work with the 1948 group had already been closed no change could be made in the examinations

given them. If an ideal experimental set up could be realized; that is, two large groups carried on at the same time, this problem would not have arisen.

The 1948 group was examined in January of 1948 after approximately one-half year of study of plane geometry by the Orleans[1/] Plane Geometry Achievement Test 1: Form A for the first half year. This consisted of five parts: geometrical reasons, computation, completing proofs, diagrams, and original proofs. The author states: "The validity of the test is shown by the fact that the correlation between test scores and teachers' school marks for ten different teachers vary from .66 to .88, the median being .81. The high validity is due in a large part to the fact that each test covers the work of only one semester and, therefore, covers it thoroughly. It is also due to the analytical organization of the test and the fact that its content and organization are aimed at the work taught in the usual geometry class."
The authors also go into a discussion of how the various forms of the test were standardized, and state "The reliability of the test is shown by the correlations between Form A and Form B for ten teachers vary from .77

1/ Joseph B. Orleans and Jacob S. Orleans, Orleans Plane Geometry Achievement Test, (World Book Company, New York, 1929)

to .87 for Test 1, with a median correlation of .85; and from .58 to .80 for Test 2, with a median correlation of .71." Percentile norms are available based on 3500 cases.

The final examination for the 1948 group given in June of 1948 was determined by the practice of the school where the experiment was carried on. Cooperative Tests of the American Council on Education are required in all subjects that tests are available for. The plane geometry classes were examined by the Cooperative Plane Geometry Test[1], Revised Series Form R, by Long, Siceloff, and Spaney. This test consists of three parts: Part one, thirty true-false statements; Part two, twenty multiple choice problems with five answers to select from; Part three, fifteen more difficult multiple choice problems with five answers to select from. Norms were available based on 9000 students from ninety schools.

The following comments covering the tests chosen for use in the experiment were taken from The Third Mental Measurements Yearbook[2]: In a review of the Orleans Plane Geometry Achievement Test, Fawcett[3] states

1/ John A. Long, L.P.Siceloff, and Emma Spaney, Cooperative Plane Geometry Test, Revised Series Form R, (Cooperative Test Service, New York, 1941)

2/ Oscar K. Buros, The Third Mental Measurements Yearbook, Rutgers University Press, New Brunswick, 1949)

3/ Harold P. Fawcett, Review of Orleans Plane Geometry Achievement Test and Cooperative Test, The Third Mental Measurements Yearbook,(Rutgers Univ. Press, New Brunswick, 1949), pp 357-362

"The test measures the factual content of geometry but ignores the larger values associated with the nature of a proof." His comment on the Cooperative Plane Geometry Test Forms R, S, and T was that they cover the facts of geometry and measure them well but, like the Orleans Test, there is not enough on the nature of a proof.

Oakley[1/] in his review of the Cooperative Plane Geometry Tests says "the test adequately covers material usually given in a plane geometry course and is well designed with respect to range and difficulty.

Because of the fact that different tests were given at the half year and the end of the year, it was necessary in order to compare the two scores to determine the gain in achievement to convert the scores on both tests to T-scores. T-scores are expressed in the same units and with respect to the same zero point and are equal throughout the scale. Hence T-scores from different tests are directly comparable and may be averaged or combined by simple addition.

1/ C.O.Oakley, Review of Cooperative Plane Geometry Tests, Forms R, S, T., The Third Mental Measurements Yearbook, (Rutgers University Press, New Brunswick, 1949) 357-362

The following Tables in Appendix A contain the data and graphs used in comparing the achievement of the two groups:

Table 9. Semi-Final and Final Test Scores for 1948 and 1949 Groups.

Table 10. Conversion of 1948 Semi-Final Scores to Sigma Scale and T-Scores.

Table 11. Conversion of 1949 Semi-Final Scores to Sigma Scale and T-Scores.

Table 12. Conversion of 1948 Final Scores to Sigma Scale and T-Scores.

Table 13. Conversion of 1949 Final Scores to Sigma Scale and T-Scores.

Table 14. Significance of the Differences between the Means on Semi-Final and Final Test Scores.

Table 15. Comparison of Scores and $\text{Sigma}_D$ for Upper Halves of Groups on Semi-Final Tests.

Table 16. Comparison of Scores and $\text{Sigma}_D$ for Lower Halves of Groups on Semi-Final Tests.

Table 17. Comparison of Scores and $\text{Sigma}_D$ for Upper Halves of Groups on Final Tests.

Table 18. Comparison of Scores and $\text{Sigma}_D$ for Lower Halves of Groups on Final Tests.

Table 19. Graphs of Achievement on Semi-Final Tests for 1948 and 1949 Groups.

Table 20. Graphs of Achievement on Final Tests for 1948 and 1949 Groups.

# CHAPTER III

## GENERAL SUMMARY

In summing up, let us first consider the two groups with which we experimented. Tables 1 and 2 of Appendix B were developed with the idea of showing the relationship of the groups used in the experiment to the larger groups of which they were members. The curve for the 1948 eleventh grade shows a median Intelligence Quotient of 101 for the seventy-six members of the grade with a median Intelligence Quotient of 107 for the plane geometry group. Of the frequency groups above the median, the plane geometry group contained a majority of the members of the entire class and below the median a much smaller membership of the entire class was found in each frequency group of the plane geometry class. This was also true for the 1949 geometry groups as compared to the 1949 eleventh grade.

These groups then can be said to show a pattern which could reasonably be expected to appear for all plane geometry groups when compared to the larger grades of which they are a part and contain most of the members of the higher Intelligence Quotient frequency groups and fewer members of the lower Intelligence Quotient frequency groups.

The attempt to equate the groups by pairing of individuals has been discussed at length in the previous

chapter; and while the same number of pairs (thirty-five) were plotted in Table 3 of Appendix A as the number of individuals kept when finally equated by groups, a glance at the summaries given on Table 3 shows a decided advantage for the 1949 group when paired with the 1948 group.

In attempting to equate by groups (see Table 4, Appendix A), it was first found that in keeping all members of each geometry group a difference of 7.48 between the means and in favor of the 1949 group was found. This difference divided by the standard error of the difference gave a ratio of 2.71 and is interpreted as a significant difference between the means. After several attempts, results of which are given in Tables 3, 4, and 5 of Appendix B, of dropping higher members of one group and lower members of the other group, a group of thirty-five members in each experimental group was chosen. The difference between the means divided by the $Sigma_D$ (see Table 5, Appendix A) gave a ratio of .78. Garrett[1] says "It is customary to take a difference divided by $Sigma_D$ of 3.0 as indicative of a significant difference (virtual certainty) since there is only about one chance

1/ Henry E. Garrett, Statistics in Psychology and Education, (New York, Longmans, Green and Co., 1940), p. 213

in 1000 that a difference of +3 Sigma will arise when the true difference is zero." Following this line of reasoning our ratio of .78 is about one-quarter of what it should be, namely, 3.0, to insure a significant difference and from Garrett[1/] Table 34 entering at .80 we find the chances 79 in 100 that the true difference is greater than zero.

While the main factor on which the groups were equated was Intelligence Quotient, the following sub-factors were checked: chronological age; previous mathematics marks; three scores from the Boston University Cooperative Test Service Vocational Battery (Problem Solving, Reading Comprehension, and Spatial Relations). These have been summarized in Chapter II in a Table of Equivalence Factors. In this table the following differences of the means are noted: chronological age, two months in favor of 1948 groups; average of previous mathematics marks, no difference in letter grades assigned; problem solving ability, no difference in mean scores; reading comprehension, no difference in mean scores; spatial relations, six points in favor of 1948 group. On all of these except the last, the differences are so small as to be considered negligible. The final sub-factor, spatial relations, where a greater difference

1/ Henry E. Garrett, Statistics in Psychology and Education, (New York, Longmans, Green and Co., 1940), Table 34, p 213

existed, might well be the deciding factor which influenced the results in favor of the 1948 group.

The data used in comparing the achievements of the two groups was obtained from the scores on the semi-final and final tests given to both groups and presented in Table 9 of Appendix A. In order to facilitate the comparison, all scores on the semi-final tests and final tests were converted to Sigma scale and T-scores and are contained in Tables 10 through 13.

Tables 19 and 20 show respectively graphs of 1948 and 1949 group achievement on semi-final tests and 1948 and 1949 group achievement on final tests. A study of these graphs brought up the question as to whether either of the methods used showed any significant difference in the halves of the group with high ability and also in the halves with low ability and led to the development of Tables 15 through 18.

The question is answered by a study of these tables which show that while slight differences exist in favor of the upper and lower halves of the 1948 group over the corresponding halves of the 1949 group, they are not significant.

Table 14 of Appendix A contains the calculation of the significance of the difference between the means on the semi-final and final tests. While some difference

existed between the means in favor of the 1948 group on both tests, the difference divided by the sigma of the difference giving ratios of 1.8 on the semi-final tests and 1.1 on the final tests indicates that these differences are not significant. The ratio of D to $\text{Sigma}_D$ for the final test of 1.1 was about one-third of what it should be; namely, 3.0 to indicate a significant difference, and that of the semi-final of 1.8 about three-fifths of what it should be. These ratios using Garrett[1/] Table 34 show that the chances are 86 in 100 that the difference is significant on the final test and 96 in 100 that the difference is significant on the semi-final test.

The table on the following page summarizes the calculations of $\text{Sigma}_D$ for Tables 14 through 18.

The method used in this table for summarizing the results of the experiment was discovered first in a research paper by Hunziker and Douglass[2/] entitled, The Relative Effectiveness of a Large Unit Plan of Supervised Study and the Daily Recitation Method in the Teaching of Algebra and Geometry. It was also used extensively in a

1/ Henry E. Garrett, Statistics in Psychology and Education, New York, Longmans, Green and Co., 1940), p. 213

2/ C.W.Hunziker and H.R.Douglass, The Relative Effectiveness of a Large Unit Plan of Supervised Study and the Daily Recitation Method in the Teaching of Algebra and Geometry, The Mathematics Teacher (March 1937), Vol. 30, No. 3, pp. 122-124

Table II. Summary of Tables 10 - 18

| Test | 1948 Mean | Group Sigma | 1949 Mean | Group Sigma | Difference of Means | $Sigma_D$ | $\frac{D}{Sigma_D}$ | Chances in 100 |
|---|---|---|---|---|---|---|---|---|
| Semi-final | 45 | 13.1 | 39 | 14.7 | 6 | 3.3 | 1.8 | 96 |
| Final | 58 | 11.2 | 55 | 11.3 | 3 | 2.7 | 1.1 | 86 |
| Upper Half Semi-final | 51 | 9.7 | 49 | 10.5 | 2 | 3.5 | .58 | 73 |
| Lower Half Semi-final | 37 | 12.3 | 30 | 12.3 | 7 | 4.2 | 1.6 | 94 |
| Upper Half Final | 63 | 10.5 | 62 | 7.3 | 1 | 3.0 | .33 | 64 |
| Lower Half Final | 52 | 9.8 | 49 | 9.2 | 3 | 3.2 | .93 | 83 |

The above Data: Lines one and two from Appendix A, Tables 10 - 14; lines three through six from Tables 15 - 18.

master's thesis by Nielsen[1] entitled, Permanent Outcomes from Teaching Plane Geometry by Two Different Methods.

One of the original premises of this paper was to compare the achievement of the two groups by the gain in achievement from semi-final to final tests. This I attempted to do graphically, but the results were not satisfactory for the purpose of drawing conclusions. Upon the suggestion of the First Reader, the table on the following page was worked up wherein the means and sigmas of the combined groups, 1948 and 1949, were presented for both tests.

This now gave a larger group with which to compare the experimental groups. Since there were the same number of pupils in each group, the mean of the combined group was easy to obtain. The sigma of the combined group was obtained by the following formula from Garrett[2]:

$$\text{Sigma}_{\text{Combined}} = \sqrt{\frac{N_1\ (\text{Sigma}^2_1 + d^2_1) + N_2\ (\text{Sigma}^2_2 + d^2_2)}{N}}$$

Dividing the difference between the group means and the combined mean by the sigma of the combined group,

1/ M.R.Nielsen, Permanent Outcomes from Teaching Plane Geometry by Two Different Methods, Master's Thesis, Iowa State College, 1938.

2/ Henry E. Garrett, Statistics in Psychology and Education, (New York, Longmans, Green and Co., 1940), p. 192

Table III. Comparison of Gains in Achievement

From Semi-Final to Final Tests

| | Semi-Final | Sigma Scale | Final | Sigma Scale | Gain |
|---|---|---|---|---|---|
| $\text{Mean}_{48}$ | 45 | +.21 | 58 | +.09 | -.12 |
| $\text{Sigma}_{48}$ | 13.1 | | 11.2 | | |
| $\text{Mean}_{49}$ | 39 | -.21 | 55 | -.18 | +.03 |
| $\text{Sigma}_{49}$ | 14.7 | | 11.3 | | |
| $\text{Mean}_{C}$ | 42 | | 57 | | |
| $\text{Sigma}_{C}$ | 14.3 | | 11.4 | | |

Calculation needed for combined group in above table:

$$\text{Semi-Final Mean} = \frac{45 + 39}{2} = 42$$

$$\text{Final Mean} = \frac{58 + 55}{2} = 56.5 = 57$$

$$\text{Semi-Final Sigma}_{\text{Combined}} = \sqrt{\frac{35\,(13.1^2 + 3^2) + 35\,(14.7^2 + (-3)^2)}{70}}$$

$$\text{Sigma}_{\text{Combined}}\ (\text{Semi-Final}) = 14.3$$

$$\text{Final Sigma}_{\text{Combined}} = \sqrt{\frac{35\,(11.2^2 + 1^2) + 35(11.3^2 + (-2)^2)}{70}}$$

$$\text{Sigma}_{\text{Combined}}\ (\text{Final}) = 11.4$$

we obtained ratios which were used for comparison of the groups.

On the semi-final test, the 1948 group was .21 sigma above the mean and the 1949 group .21 sigma below the mean. On the final test, the 1948 group was only .13 sigma above the mean and the 1949 group was .13 sigma below the mean. The last column is interpreted to indicate that the 1948 group showed a slight loss while the 1949 group showed a slight gain.

An interesting sidelight of the experiment was the development of the Classification Table of Types of Plane Geometry Textbooks (Table 7, Appendix A). While there are only a few books included in each of the first three groups, about half of these are now published as revised or second editions. Note particularly the copyright dates of the first three groups which, with the exception of the first book in group three, first appeared about 1938-1940. Group four with by far the greatest number of books represented in it covers a period of time from 1915-1948. A closer examination of these textbooks and any others which might be available might well be the subject of another service paper.

## CHAPTER IV

## CONCLUSION AND RECOMMENDATIONS

Results of this experiment show no significant difference between the formal and informal methods relative to achievement in factual information and skills in reasoning about geometric situations.

The informal method required less time than the formal method to cover the units of work in the plane geometry course. The time saved was used for units in space geometry and loci constructions, topics which have invariably been left out of the usual course in plane geometry for lack of time.

Comparing the groups as a whole, there was a slight difference in favor of the group using the formal method; statistically this was not found to be significant.

Comparing the upper and lower halves of the groups graphically (Tables 19 and 20), there was a difference in favor of the informal method for the upper halves of the groups in both semi-final and final tests. This contrasted with the lower halves of the groups where the difference was in favor of the formal method on both semi-final and final tests. All differences between whole and half groups were found to be in favor of the 1948 group, but

were not significant. We may well conclude that despite the material left out (proof of propositions) there was no loss in achievement by the informal group whose achievement was very close to that of the formal group.

This study does not show that either the formal method or the informal method has any measurable difference in effect upon pupil achievement, and while no statement can be made on the statistical evidence as to which is the better method of teaching plane geometry; nevertheless, the writer feels that the saving in time alone gives the informal method an advantage over the informal method.

No decision can be made from the statistical results of this experiment as to the choice of type of textbook to be used, but the writer believes that an informal-type textbook would increase the amount of time available to be used for additional units.

The results of this experiment as expressed in the conclusions are not what the writer expected. While holding a slight prejudice in favor of the informal method, every effort was made to keep this from influencing the experiment and certainly the results bear this out.

Let us next consider whether the results of this experiment are reasonable or not. A comparison of the

graphs of Tables 19 and 20 in Appendix A show very clearly that the same pattern of achievement existed on both the semi-final and final tests. The difference between the two curves on each table are small and might well be considered to indicate no difference in achievement between the groups regardless of the method used.

There are many factors which influence an experiment of this type, not all of which can be brought into the equating process. I have in mind an incident which happened with the 1949 group who were taking their final tests during a period of exceptionally hot, humid weather. This might well be considered to have influenced their achievement on this test.

It is recommended that this experimental paper be used as a pilot for a future experiment with the following factors being carefully checked:

1. Larger groups are needed for better equating.

2. Both methods should be used in the same school year with rotation of groups.

3. A pre-test should be given to determine what geometrical factors and skills the groups possessed at the beginning of the experiment.

4. Use the same test or other forms of the same test for the three test periods; pre-test, semi-final, and final.

5. Use an informal-type book (see Table 7, Appendix A) with groups using the informal method.

# APPENDIX A

Table 1. A Comparison of Axioms and Postulates; Theorems, Corollaries, and Constructions of six often-used Textbooks. 1/

| Name of Author | Axioms and Postulates of Plane Geometry | Theorems, Corollaries and Constructions of Plane Geometry | Axioms and Postulates of Solid Geometry | Theorems and Corollaries of Solid Geometry |
|---|---|---|---|---|
| Durrell and Arnold | 38 | 196 | 11 | 153 |
| Nyberg | 52 | 198 | 15 | 127 |
| Otis Clark | 38 | 165 | 9 | 114 |
| Seymour | 48 | 218 | | |
| Smith, Foberg, and Reeve | 33 | 162 | 16 | 59 |
| Wells, Hart | 29 | 231 | | |
| Average | 40 | 195 | 12 | 131 |
| This Study | 32 | 30 | 4 | 12 |

Note: There may be errors in the above count because of cases where it was not clear whether or not the author intended statement for a definition, a postulate, or a theorem. Some authors also used principles, properties, and problems; and again it was not clear what these were intended for.

1/ H.C.Christofferson, Geometry Professionalized for Teachers, George Banta Publishing Company, Menasha, Wisconsin, 1933, Table LV, p. 43

Table 2. Suggested Places in Plane Geometry in which we can Shorten, Eliminate, or Postulate to Allow Time to Introduce Space Concepts. 1/

1. We may omit, to be included later, the theorem, "The exterior angle of a triangle is greater than either opposite interior angle."

2. It is suggested that we postulate the theorem "One and only one line can be drawn from (at) a given point perpendicular to a given line."

3. If the hypotenuse - angle congruency proposition is proved by superposition, it might be postulated.

4. It might be possible to postulate the theorem "If two angles have their sides respectively perpendicular, they are either equal or supplementary."

5. The theorem "The sum of the angles of a polygon of n sides is (n-2) straight angles" may be postulated or proved informally.

6. In certain texts it will be found possible to do some reversing and some combining of the theorems on the properties of a parallelogram.

7. In connection with inequalities in a triangle or a circle, certain theorems and corollaries might be postulated.

8. Group the theorems concerning interdependence of central angles, their arcs and chords; interdependence of diameters and perpendicular bisectors or chords; interdependence of tangents and radii to the points of contact.

9. The concurrency propositions, having been intuitively established in the introduction, may be postulated.

10. Eliminate harmonic division of a line segment as a separate treatment.

11. Substitute the algebraic for the geometric proof of the Pythagorean Theorem.

Table 2. (continued)

12. Eliminate the generalized Pythagorean Theorems referring to acute and obtuse triangles.

13. Eliminate the proof of Hero's formula or make it a matter for class discussion only.

14. Postulate the theorem "If two polygons are similar, they can be divided into triangles which are similar and similarly placed" and its converse.

15. Postulate the continuity of the Pythagorean Theorem as applied to similar polygons constructed upon the three sides of a right triangle.

1/ From Report of Second Annual Purdue Mathematics Workshop Committee on first six weeks in Plane Geometry, June 16 - 28, 1947

Table 3. Chart Showing Pairs of 1948 and 1949 Plane Geometry Students, Plotting Intelligence Quotients versus Average of Mathematics Marks

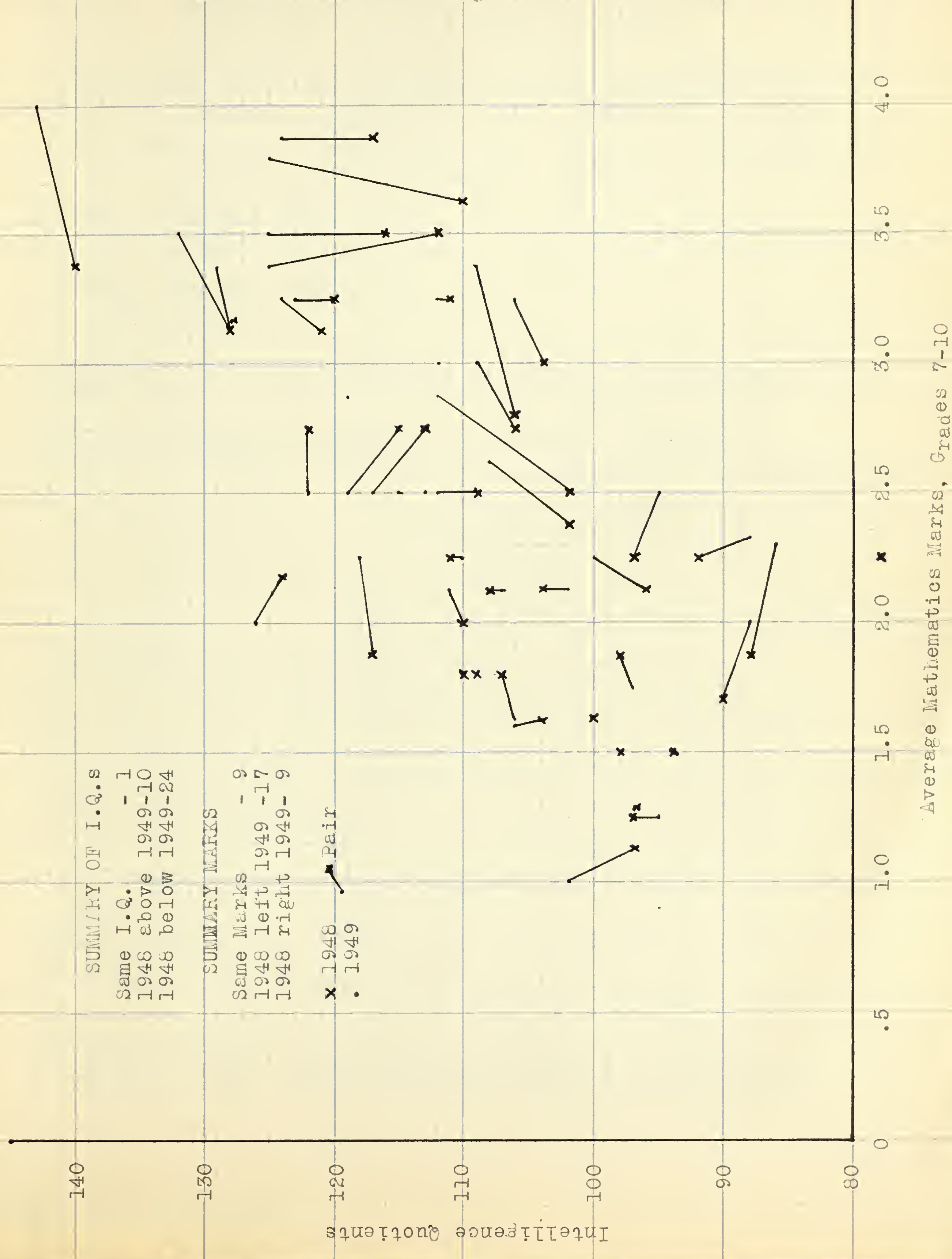


Table 4. Significance of the Difference Between the Means - Unequated Groups

| | | |
|---|---|---|
| 1948 - 44 students | Mean 104.56 = 105 | Sigma = 12.05 = 12.1 |
| 1949 - 39 students | Mean 112.05 = 112 | Sigma = 11.98 = 12.0 |

$$\text{Sigma}_{M_{48}} = \frac{12.1}{\sqrt{44}} = \frac{12.1}{6.63} = 1.91$$

$$\text{Sigma}_{M_{49}} = \frac{12.0}{\sqrt{39}} = \frac{12.0}{6.25} = 1.92$$

Standard error of the difference between two uncorrelated means.

$$\text{Sigma}_D \text{ or } \text{Sigma}_{M_{49}} - \text{Sigma}_{M_{48}} = \sqrt{\text{Sigma}_{M_{49}}^{\ 2} + \text{Sigma}_{M_{48}}^{\ 2}}$$

$$= \sqrt{1.91^2 + 1.92^2}$$

$$= \sqrt{7.3345}$$

$$\text{Sigma}_D = 2.71$$

Significance of the difference between the means.

$M_{49} - M_{48} = 7.48$. Chances are 68 in 100 that the difference of 7.48 does not differ from the true difference by more than $\pm$ 2.71 and chances are 99 in 100 that the difference of 7.48 does not differ from the true difference by more than $\pm 3 \times 2.71 = \pm 8.13$ and lies between -.65 and +15.61.

$\frac{D}{\text{Sigma}_D} = \frac{7.48}{2.71} = 2.76$. Using 2.76, from Garrett[1], chances are 99.72 (100) or almost virtual certainty that the difference is greater than zero between the two groups.

1/ Henry E. Garrett, Statistics in Psychology and Education, Longmans, Green and Co., New York, 1940, Table 34, p. 213

Table 5. Significance of the Difference Between the Means - Equated Groups

| | | |
|---|---|---|
| 1948 - 35 students | Mean 107.8 = 108 | Sigma = 10.24 = 10.2 |
| 1949 - 35 students | Mean 109.7 = 110 | Sigma = 10.09 = 10.1 |

$$\text{Sigma}_{M_{48}} = \frac{10.2}{\sqrt{35}} = \frac{10.2}{5.91} = 1.73$$

$$\text{Sigma}_{M_{49}} = \frac{10.0}{\sqrt{35}} = \frac{10.1}{5.92} = 1.70$$

Standard error of the difference between two uncorrelated means.

$$\text{Sigma}_D \text{ or } \text{Sigma}_{M_{49}} - \text{Sigma}_{M_{48}} = \sqrt{\text{Sigma}_{M_{49}}^{\;2} + \text{Sigma}_{M_{48}}^{\;2}}$$

$$= \sqrt{1.73^2 + 1.70^2}$$

$$= \sqrt{5.8829}$$

$$\text{Sigma}_D = 2.43$$

Significance of the difference between the means.

$M_{49} - M_{48} = 1.9$. Chances are 68 in 100 that the difference of 1.9 does not differ from the true difference by more than $\pm$ 2.43 and chances are 99 in 100 that the difference of 1.9 does not differ from the true difference by more than $\pm$ 3 x 2.43 = $\pm$ 7.29 and lies between -5.39 and +9.19. $\frac{D}{\text{Sigma}_D} = \frac{1.9}{2.42} = .78$. Using .80, from Garrett1/, chances are 79 in 100 that the true difference is greater than zero. Since the lower limit is negative (-5.38), there is some chance that the true difference is less than zero.

1/ Henry E. Garrett, Statistics in Psychology and Education, Longmans, Green and Co., New York, 1940, Table 34, p. 213

Table 6. Theorems Postulated from Welchons & Krickenberger Plane Geometry

1. If two triangles have two sides and the included angle of one equal respectively to two sides and the included angle of the other, the triangles are congruent.

2. If two triangles have two angles and the included side of one equal respectively to two angles and the included side of the other, the triangles are congruent.

3. If two triangles have three sides of one equal respectively to three sides of the other, the triangles are congruent.

4. An exterior angle of a triangle is greater than either non-adjacent interior angle.

5. If two lines form equal alternate interior angles with a transversal, the lines are parallel.

6. If two parallels are cut by a transversal, the alternate interior angles are equal.

7. In a circle or in equal circles equal central angles have equal arcs.

8. In a circle or in equal circles equal arcs have equal central angles.

9. If a line is tangent to a circle, it is perpendicular to the radius drawn to the part of contact.

10. If a line is perpendicular to a radius at the point on a circle, the line is tangent to the circle.

11. If a line is parallel to one side of a triangle and intersects the other two sides, it divides these sides proportionally.

12. If a line divides two sides of a triangle proportionally, it is parallel to the third side.

13. If two triangles have two angles of one equal respectively to two angles of the other, the triangles are similar.

Table 6. (continued)

14. If two triangles have their sides respectively proportional, they are similar.

15. A circle can be circumscribed about any regular polygon.

16. If two triangles have two sides of one equal respectively to two sides of the other and the included angle of the first greater than the included angle of the second, the third side of the first is greater than the third side of the second.

17. If two triangles have two sides of one equal respectively to two sides of the other and the included angle of the first greater than the included angle of the second, the third side of the first is greater than the third side of the second, the angle opposite the third side of the first is greater than the angle opposite the third side of the second.

18. In a circle or equal circles the greater of two central angles has the greater arc.

19. In a circle or equal circles the greater of two unequal arcs has the greater central angle.

20. In a circle or in equal circles the greater of two unequal chords has the greater arc.

21. In a circle or in equal circles the greater of two unequal arcs has the greater chord.

Table 7. Classification of Plane Geometry Textbooks

| Title | Author | Publisher | Copyright |
|---|---|---|---|
| <u>Group I Many assumed theorems; few formal proofs</u>. | | | |
| <u>Plane Geometry</u> | Major | Scribner | 1938 |
| <u>Today's Geometry</u> | Reichgott & Spiller | Prentice-Hall | 1938 |
| <u>Basic Geometry</u> | Birkhoff & Beatley | Scott, Foresman | 1940-41 |
| <u>Clear Thinking</u> | Schnell & Crawford | Harpers | 1938-43 |
| <u>Group II Postulation of group of about twenty theorems</u>. | | | |
| <u>Plane Geometry and Its Reasoning</u> | Barber & Hendrix | Harcourt, Brace | 1937 |
| <u>A New Geometry for Secondary Schools</u> | Herberg & Orleans | D.C.Heath | 1940-48 |
| <u>Group III Postulation of Congruent Triangles</u> | | | |
| <u>Senior Math, Book II of Unified Math Series</u> | Breslich | Univ. of Chicago Press | 1910,16, 23,27 |
| <u>Plane Geometry</u> | Seymour & Smith | Macmillan | 1941 |
| <u>Modern School Geometry</u> | Schorling, Clark, Smith | World Book Co. | 1938,43, 48 |

Table 7 (continued)

| Title | Author | Publisher | Copyright |
|---|---|---|---|
| Group IV Formal proofs; no postulation of theorems. | | | |
| New Plane Geometry | Robbins | American | 1915 |
| Modern Plane Geometry | Stone-Mallory | Sanborn | 1929 |
| New Plane Geometry | Durrell & Arnold | Merrill | 1916,17 24,30 |
| Plane Geometry | Seymour | American | 1925,31 |
| Progressive Plane Geometry | Wells & Hart | D.C.Heath | 1935 |
| Plane Geometry and Its Uses | Mirick, Newell & Harper | Row-Peterson | 1935 |
| Plane Geometry and Related Subjects | Breslich | Laidlow | 1910,16, 23,27, 35 |
| Plane Geometry | Schulze-Sevenoak-Stone | Macmillan | 1936 |
| Plane Geometry | Morgan-Foberg-Breckenridge | Houghton Mifflin | 1937 |
| Plane Geometry of Purposeful Math Series | Breslich | Laidlow | 1938 |
| New Plane Geometry | Stone-Mallory | Sanborn | 1938 |
| Plane Geometry | Welchons & Krickenberger | Ginn | 1933,38 40,43 |

Table 7 (continued)

| Title | Author | Publisher | Copyright |
|---|---|---|---|
| Fundamentals of Plane Geometry | Nyberg | American | 1944 |
| Plane Geometry | Avery | Allyn & Bacon | 1947 |
| New Plane Geometry | Smith-Marino | Merrill | 1948 |

Table 8. Timetable of Work - 1948 Group vs. 1949 Group

| | Unit | Days taken by 1948 Group | Days taken by 1949 Group | Difference |
|---|---|---|---|---|
| 1. | Introduction | 15 | 11 | +4 |
| 2. | Triangles | 21 | 19 | +2 |
| 3. | Parallels and Perpendiculars | 27 | 19 | +8 |
| 4. | Constructions | 5 | 6 | -1 |
| 5. | Polygons | 21 | 19 | +2 |
| | Half-year Totals | 89 | 74 | +15 |
| 6. | Area of Polygons | 19 | 20 | -1 |
| 7. | Circles, Angles, and Arcs | 13 | 12 | +1 |
| 8. | Measurement of Angles and Arcs | 12 | 12 | 0 |
| 9. | Loci | 13 | 20 | -7 |
| 10. | Proportion | 11 | 9 | +2 |
| 11. | Similar Polygons | 13 | 13 | 0 |
| 12. | Regular Polygons and the Circle | 5 | 5 | 0 |
| | Totals | 175 | 165 | -5 |
| x | Space Geometry | 0 | 10 | -10 |

Table 9. Semi-Final and Final Test Scores for 1948 and 1949 Groups

| 1948 Group | | | 1949 Group | | |
|---|---|---|---|---|---|
| IQ | Semi-final | Final | IQ | Semi-final | Final |
| 140 | 57 | 68 | 125 | 59 | 67 |
| 128 | 56 | 71 | 125 | 65 | 72 |
| 128 | 62 | 68 | 125 | 62 | 68 |
| 124 | 34 | 54 | 124 | 67 | 73 |
| 122 | 34 | 67 | 124 | 55 | 50 |
| 121 | 53 | 66 | 123 | 37 | 58 |
| 120 | 52 | 65 | 122 | 42 | 58 |
| 117 | 43 | 64 | 119 | 26 | 61 |
| 117 | 60 | 70 | 118 | 36 | 56 |
| 116 | 66 | 71 | 117 | 36 | 55 |
| 115 | 44 | 50 | 115 | 52 | 61 |
| 113 | 54 | 49 | 113 | 58 | 64 |
| 112 | 61 | 79 | 112 | 52 | 63 |
| 111 | 50 | 68 | 112 | 45 | 71 |
| 111 | 59 | 70 | 112 | 44 | 67 |
| 110 | 62 | 68 | 112 | 44 | 51 |
| 110 | 43 | 50 | 111 | 33 | 53 |
| 110 | 42 | 41 | 110 | 42 | 61 |
| 109 | 28 | 50 | 109 | 50 | 56 |
| 109 | 35 | 26 | 109 | 50 | 63 |
| 108 | 25 | 57 | 108 | 33 | 49 |
| 107 | 43 | 49 | 107 | 39 | 55 |
| 106 | 49 | 55 | 106 | 33 | 53 |
| 106 | 58 | 67 | 106 | 17 | 29 |
| 104 | 58 | 64 | 106 | 15 | 35 |
| 104 | 33 | 60 | 102 | 31 | 53 |
| 104 | 24 | 51 | 102 | 28 | 54 |
| 102 | 55 | 62 | 100 | 38 | 35 |
| 102 | 26 | 43 | 98 | 22 | 53 |
| 100 | 31 | 49 | 98 | 14 | 49 |

Table 9. (continued)

| 1948 Group | | | 1949 Group | | |
|---|---|---|---|---|---|
| IQ | Semi-final | Final | IQ | Semi-final | Final |
| 98 | 34 | 41 | 97 | 42 | 55 |
| 98 | 34 | 53 | 97 | 12 | 43 |
| 98 | 24 | 48 | 95 | 13 | 48 |
| 97 | 24 | 48 | 95 | 38 | 57 |
| 97 | 53 | 63 | 86 | 39 | 23 |
| Means | | | | | |
| 108 | 45 | 58 | 110 | 39 | 55 |
| Sigmas | | | | | |
| 102 | 13.1 | 11.2 | 10.1 | 14.7 | 11.3 |

Table 10. Conversion of 1948 Semi-Final Scores to Sigma Scale and T-Scores.

| Score | d | $d^2$ | Sigma Scale | T-Score |
|---|---|---|---|---|
| 66 | 21 | 441 | 1.62 | 66.2 |
| 62 | 17 | 289 | 1.31 | 63.1 |
| 62 | 17 | 289 | 1.31 | 63.1 |
| 61 | 16 | 256 | 1.23 | 62.3 |
| 60 | 15 | 225 | 1.15 | 61.5 |
| 59 | 14 | 196 | 1.08 | 60.8 |
| 58 | 13 | 169 | 1.00 | 60.0 |
| 58 | 13 | 169 | 1.00 | 60.0 |
| 57 | 12 | 144 | .92 | 59.2 |
| 56 | 11 | 121 | .85 | 58.5 |
| 55 | 10 | 100 | .77 | 57.7 |
| 54 | 9 | 81 | .69 | 56.9 |
| 53 | 8 | 64 | .62 | 56.2 |
| 53 | 8 | 64 | .62 | 56.2 |
| 52 | 7 | 49 | .54 | 55.4 |
| 50 | 5 | 25 | .38 | 53.8 |
| 49 | 4 | 16 | .31 | 53.1 |
| 44 | -1 | 1 | -.08 | 49.2 |
| 43 | -2 | 4 | -.15 | 48.5 |
| 43 | -2 | 4 | -.15 | 48.5 |
| 43 | -2 | 4 | -.15 | 48.5 |
| 42 | -3 | 9 | -.23 | 47.7 |
| 35 | -10 | 100 | -.77 | 42.3 |
| 34 | -11 | 121 | -.85 | 41.5 |
| 34 | -11 | 121 | -.85 | 41.5 |
| 34 | -11 | 121 | -.85 | 41.5 |
| 34 | -11 | 121 | -.85 | 41.5 |
| 33 | -12 | 144 | -.92 | 40.8 |
| 31 | -14 | 196 | -1.08 | 39.2 |
| 28 | -17 | 289 | -1.31 | 36.9 |

Table 10. (continued)

| Score | d | $d^2$ | Sigma Scale | T-Score |
|---|---|---|---|---|
| 26 | -19 | 361 | -1.31 | 36.9 |
| 25 | -20 | 400 | -1.54 | 34.6 |
| 24 | -21 | 441 | -1.62 | 33.8 |
| 24 | -21 | 441 | -1.62 | 33.8 |
| 24 | -21 | 441 | -1.62 | 33.8 |
| 35 \| 1566 | | 35 \| 6017 | | |
| 44.74 | | 171.91 | | |

Mean = 45 Sigma = 13.1

Table 11. Conversion of 1949 Semi-Final Scores to Sigma Scale and T-Scores

| Score | d | $d^2$ | Sigma Scale | T-Score |
|---|---|---|---|---|
| 67 | 28 | 784 | 1.9 | 69.0 |
| 65 | 26 | 676 | 1.77 | 67.7 |
| 62 | 23 | 529 | 1.56 | 65.6 |
| 59 | 20 | 400 | 1.36 | 63.6 |
| 58 | 19 | 361 | 1.29 | 62.9 |
| 55 | 16 | 256 | 1.09 | 60.9 |
| 52 | 13 | 169 | .95 | 58.5 |
| 51 | 12 | 144 | .86 | 58.2 |
| 50 | 11 | 121 | .75 | 57.5 |
| 50 | 11 | 121 | .75 | 57.5 |
| 45 | 6 | 36 | .41 | 54.1 |
| 44 | 5 | 25 | .34 | 53.4 |
| 44 | 5 | 25 | .34 | 53.4 |
| 42 | 3 | 9 | .20 | 52.0 |
| 42 | 3 | 9 | .20 | 52.0 |
| 42 | 3 | 9 | .20 | 52.0 |
| 39 | 0 | | 0 | 50. |
| 39 | 0 | | 0 | 50. |
| 38 | -1 | 1 | -.07 | 49.3 |
| 38 | -1 | 1 | -.07 | 49.3 |
| 37 | -2 | 4 | -.14 | 48.6 |
| 36 | -3 | 9 | -.20 | 48.0 |
| 36 | -3 | 9 | -.20 | 48.0 |
| 33 | -6 | 36 | -.41 | 45.9 |
| 33 | -6 | 36 | -.41 | 45.9 |
| 33 | -6 | 36 | -.41 | 45.9 |
| 31 | -8 | 64 | -.54 | 44.6 |
| 28 | -11 | 121 | -.75 | 42.5 |
| 26 | -13 | 169 | -.95 | 40.5 |
| 22 | -17 | 289 | -1.16 | 38.4 |

Table 11. (continued)

| Score | d | $d^2$ | Sigma Scale | T-Score |
|---|---|---|---|---|
| 17 | -22 | 484 | -1.50 | 35.0 |
| 15 | -24 | 576 | -1.63 | 33.7 |
| 14 | -25 | 625 | -1.7 | 33.0 |
| 13 | -26 | 676 | -1.77 | 32.3 |
| 12 | -27 | 729 | -1.84 | 31.6 |
| 35 \| 1368 | | 35 \| 7539 | | |
| 39.08 | | 215.4 | | |

Mean = 39 Sigma = 14.7

Table 12. Conversion of 1948 Final Scores to Sigma Scale and T-Scores

| Score | d | $d^2$ | Sigma Scale | T-Score |
|---|---|---|---|---|
| 79 | 21 | 441 | 19.1 | 69.1 |
| 71 | 13 | 169 | 11.8 | 61.8 |
| 71 | 13 | 169 | 11.8 | 61.8 |
| 70 | 12 | 144 | 10.9 | 60.9 |
| 70 | 12 | 144 | 10.9 | 60.9 |
| 68 | 10 | 100 | 9.1 | 59.1 |
| 68 | 10 | 100 | 9.1 | 59.1 |
| 68 | 10 | 100 | 9.1 | 59.1 |
| 68 | 10 | 100 | 9.1 | 59.1 |
| 67 | 9 | 81 | 8.2 | 58.2 |
| 67 | 9 | 81 | 8.2 | 58.2 |
| 66 | 8 | 64 | 7.3 | 57.3 |
| 65 | 7 | 49 | 6.4 | 56.4 |
| 64 | 6 | 36 | 5.5 | 55.5 |
| 64 | 6 | 36 | 5.5 | 55.5 |
| 63 | 5 | 25 | 4.5 | 54.5 |
| 62 | 4 | 16 | 3.6 | 53.6 |
| 60 | 2 | 4 | 1.8 | 51.8 |
| 57 | -1 | 1 | .9 | 49.1 |
| 55 | -3 | 9 | -2.7 | 47.3 |
| 54 | -4 | 16 | -3.6 | 46.4 |
| 53 | -5 | 25 | -4.5 | 45.5 |
| 51 | -7 | 49 | -6.4 | 43.6 |
| 50 | -8 | 64 | -7.3 | 42.7 |
| 50 | -8 | 64 | -7.3 | 42.7 |
| 50 | -8 | 64 | -7.3 | 42.7 |
| 49 | -9 | 81 | -8.2 | 41.8 |
| 49 | -9 | 81 | -8.2 | 41.8 |
| 49 | -9 | 81 | -8.2 | 41.8 |
| 48 | -10 | 100 | -9.1 | 40.9 |

Table 12 (continued)

| Score | d | $d^2$ | Sigma Scale | T-Score |
|---|---|---|---|---|
| 48 | -10 | 100 | -9.1 | 40.9 |
| 43 | -15 | 225 | -13.6 | 36.4 |
| 41 | -17 | 289 | -15.4 | 34.6 |
| 41 | -17 | 289 | -15.4 | 34.6 |
| 26 | -32 | 1024 | -29.0 | 21.0 |
| 35 \| 2025 | | 35 \| 4421 | | |
| 57.57 | | 126.3 | | |

Mean = 58 Sigma = 11.2

Table 13. Conversion of 1949 Final Scores to Sigma Scale and T-Scores

| Score | d | $d^2$ | Sigma Scale | T-Score |
|---|---|---|---|---|
| 73 | 18 | 324 | 16.4 | 66.4 |
| 72 | 17 | 289 | 15.5 | 65.5 |
| 71 | 16 | 256 | 14.5 | 64.5 |
| 68 | 13 | 169 | 12.8 | 61.8 |
| 67 | 12 | 144 | 10.9 | 60.9 |
| 67 | 12 | 144 | 10.9 | 60.9 |
| 64 | 9 | 81 | 8.2 | 58.2 |
| 63 | 8 | 64 | 7.3 | 57.3 |
| 63 | 8 | 64 | 7.3 | 57.3 |
| 61 | 6 | 36 | 5.5 | 55.5 |
| 61 | 6 | 36 | 5.5 | 55.5 |
| 61 | 6 | 36 | 5.5 | 55.5 |
| 58 | 3 | 9 | 2.7 | 52.7 |
| 58 | 3 | 9 | 2.7 | 52.7 |
| 57 | 2 | 4 | 1.8 | 51.8 |
| 56 | 1 | 1 | .9 | 50.9 |
| 56 | 1 | 1 | .9 | 50.9 |
| 55 | 0 | | 0 | 50.0 |
| 55 | 0 | | 0 | 50.0 |
| 55 | 0 | | 0 | 50.0 |
| 54 | -1 | 1 | -.9 | 49.1 |
| 53 | -2 | 4 | -1.8 | 48.2 |
| 53 | -2 | 4 | -1.8 | 48.2 |
| 53 | -2 | 4 | -1.8 | 48.2 |
| 53 | -2 | 4 | -1.8 | 48.2 |
| 51 | -4 | 16 | -3.6 | 46.4 |
| 50 | -5 | 25 | -4.5 | 45.6 |
| 49 | -6 | 36 | -5.5 | 44.5 |
| 49 | -6 | 36 | -5.5 | 44.5 |
| 48 | -7 | 49 | -6.4 | 43.6 |

Table 13. (continued)

| Score | d | $d^2$ | Sigma Scale | T-Score |
|---|---|---|---|---|
| 43 | -12 | 144 | -10.9 | 39.1 |
| 35 | -20 | 400 | -18.2 | 31.8 |
| 35 | -20 | 400 | -18.2 | 31.8 |
| 29 | -26 | 676 | -23.6 | 26.4 |
| 23 | -32 | 1024 | -29.1 | 20.9 |
| 35 \| 1919 | | 35 \| 4490 | | |
| 54.82 | | 128.28 | | |

Mean = 55 Sigma 11.3

Table 14. Significance of the Differences between the Means of the 1948 and 1949 Groups on the Semi-Final and Final Test Scores

A. Semi-Final Test

| | | | |
|---|---|---|---|
| 1948 | 35 Students | Mean = 45 | Sigma = 13.1 |
| 1949 | 35 Students | Mean = 39 | Sigma = 14.7 |

$\text{Sigma}_{M_{48}} = \frac{13.1}{\sqrt{35}} = 2.22$ $\quad \text{Sigma}_{M_{49}} = \frac{14.7}{\sqrt{35}} = 2.48$

$\text{Sigma}_D \text{ or } \text{Sigma}_{M_{48}} - \text{Sigma}_{M_{49}} = \sqrt{2.22^2 + 2.48^2}$

$\text{Sigma}_D = 3.3$ $\quad \frac{D}{\text{Sigma}_D} = \frac{6}{3.3} = 1.8^*$

*From Garrett Table 34, p. 213. Chances are 96 in 100

B. Final Test

| | | | |
|---|---|---|---|
| 1948 | 35 Students | Mean = 58 | Sigma = 11.2 |
| 1949 | 35 Students | Mean = 55 | Sigma = 11.3 |

$\text{Sigma}_{M_{48}} = \frac{11.2}{\sqrt{35}} = 1.90$ $\quad \text{Sigma}_{M_{49}} = \frac{11.3}{\sqrt{35}} = 1.92$

$\text{Sigma}_D \text{ or } \text{Sigma}_{M_{48}} - \text{Sigma}_{M_{49}} = \sqrt{1.90^2 + 1.92^2}$

$\text{Sigma}_D = 2.7$ $\quad \frac{D}{\text{Sigma}_D} = \frac{3.0}{2.7} = 1.1^*$

*From Garrett Table 34, p. 213. Chances are 86 in 100

Table 15. Comparison of Scores and Significance of the Difference between the Means for the Upper Halves of the Groups on Semi-Final Tests

| Upper Half of 1948 Group | | | Upper Half of 1949 Group | | |
|---|---|---|---|---|---|
| Score | d | $d^2$ | Score | d | $d^2$ |
| 66 | 15 | 225 | 67 | 18 | 324 |
| 62 | 11 | 121 | 65 | 16 | 256 |
| 62 | 11 | 121 | 62 | 13 | 169 |
| 61 | 10 | 100 | 59 | 10 | 100 |
| 60 | 9 | 81 | 58 | 9 | 81 |
| 59 | 8 | 64 | 44 | 6 | 36 |
| 56 | 5 | 25 | 52 | 3 | 9 |
| 54 | 3 | 9 | 51 | 2 | 4 |
| 53 | 2 | 4 | 45 | -4 | 16 |
| 52 | 1 | 1 | 44 | -5 | 25 |
| 52 | -1 | 1 | 44 | -5 | 25 |
| 44 | -7 | 49 | 42 | -7 | 49 |
| 43 | -8 | 64 | 42 | -7 | 49 |
| 43 | -8 | 64 | 37 | -12 | 144 |
| 42 | -9 | 81 | 36 | -13 | 169 |
| 34 | -17 | 289 | 36 | -13 | 169 |
| 34 | -17 | 289 | 33 | -16 | 256 |
| 17 ) 875 | | 17 ) 1588.0 | 17 ) 828 | | 17 ) 1881 |
| 51 | | 93.41 | 48.7 | | 110.64 |

Mean = 51 Sigma = 9.7 Mean = 49 Sigma = 10.5

$\text{Sigma}_D$ of Upper Halves

$M_{48} - M_{49} = 51 - 49 = 2.0$

$\text{Sigma}_{M_{48}} = \frac{9.7}{\sqrt{17}} = 2.34$

$\text{Sigma}_{M_{49}} = \frac{10.5}{\sqrt{17}} = 2.55$

$\text{Sigma}_D = \sqrt{2.34^2 + 2.55^2}$

$\text{Sigma}_D = 3.5$

$\frac{D}{\text{Sigma}_D} = \frac{2.0}{3.5} = .58$

From Garrett Table 34 at .60

Chances are 73 in 100

Table 16. Comparison of Scores and Significance of the difference between the Means for the Lower Halves of the Groups on Semi-Final Tests

| Lower Half of 1948 Group | | | Lower Half of 1949 Group | | |
|---|---|---|---|---|---|
| Score | d | $d^2$ | Score | d | $d^2$ |
| 58 | 22 | 441 | 50 | 20 | 400 |
| 58 | 21 | 441 | 50 | 20 | 400 |
| 55 | 18 | 324 | 42 | 12 | 144 |
| 53 | 16 | 256 | 39 | 9 | 81 |
| 49 | 12 | 144 | 39 | 9 | 81 |
| 43 | 6 | 36 | 38 | 8 | 64 |
| 35 | -2 | 4 | 38 | 8 | 64 |
| 34 | -3 | 9 | 33 | 3 | 9 |
| 34 | -3 | 9 | 33 | 3 | 9 |
| 33 | -4 | 16 | 31 | 1 | 1 |
| 31 | -6 | 36 | 28 | -2 | 4 |
| 28 | -9 | 81 | 22 | -8 | 64 |
| 26 | -11 | 121 | 17 | -13 | 169 |
| 25 | -12 | 144 | 15 | -15 | 225 |
| 24 | -13 | 169 | 14 | -16 | 256 |
| 24 | -13 | 169 | 13 | -17 | 289 |
| 24 | -13 | 169 | 12 | -18 | 324 |
| 17 ⟌634 | | 17 ⟌2569 | 17 ⟌514 | | 17 ⟌2584 |
| 37.2 | | 151.1 | 30.2 | | 152 |

Mean = 37 Sigma = 12.3 Mean = 30 Sigma = 12.3

$\text{Sigma}_D$ of Lower Halves

$M_{48} - M_{49} = 37 - 30 = 7.0$

$\text{Sigma}_{M_{48}} = \frac{12.3}{\sqrt{17}} = 2.98$

$\text{Sigma}_{M_{49}} = \frac{12.3}{\sqrt{17}} = 2.90$

$\text{Sigma}_D = \sqrt{2.98^2 + 2.98^2}$

$\text{Sigma}_D = 4.2$

$\frac{D}{\text{Sigma}_D} = \frac{7.0}{4.2} = 1.6$

From Garrett Table 34 at 1.60

Chances are 94 in 100

Table 17. Comparison of Scores and Significance of the Difference between the Means for the Upper Halves of the groups on Final Tests

| Upper Half of 1948 Group | | | Upper Half of 1949 Group | | |
|---|---|---|---|---|---|
| Score | d | $d^2$ | Score | d | $d^2$ |
| 79 | 16 | 256 | 73 | 12 | 144 |
| 71 | 8 | 64 | 72 | 11 | 121 |
| 71 | 8 | 64 | 71 | 10 | 100 |
| 70 | 7 | 49 | 68 | 6 | 36 |
| 70 | 7 | 49 | 67 | 5 | 25 |
| 68 | 5 | 25 | 67 | 5 | 25 |
| 68 | 5 | 25 | 64 | 2 | 4 |
| 68 | 5 | 25 | 63 | 1 | 1 |
| 67 | 4 | 16 | 61 | -1 | 1 |
| 66 | 3 | 9 | 31 | -1 | 1 |
| 65 | 2 | 4 | 58 | -4 | 16 |
| 64 | 1 | 1 | 58 | -4 | 16 |
| 54 | -9 | 81 | 56 | -6 | 36 |
| 50 | -13 | 169 | 55 | -6 | 36 |
| 50 | -13 | 169 | 53 | -9 | 81 |
| 49 | -14 | 199 | 51 | -11 | 121 |
| 41 | -22 | 484 | 50 | -12 | 144 |
| 17 ⌊1071 | | 17 ⌊1689 | 17 ⌊1048 | | 17 ⌊9080 |
| 63 | | 99.3 | 61.6 | | 53.41 |

Mean = 63　　Sigma = 10　　Mean = 62　　Sigma = 7.33

$\text{Sigma}_D$ of Upper Halves

$$M_{48} - M_{49} = 63 - 62 = 1.0$$

$$\text{Sigma}_{M_{48}} = \frac{10}{\sqrt{17}} = 2.41$$

$$\text{Sigma}_{M_{49}} = \frac{7.3}{\sqrt{17}} = 1.78$$

$$\text{Sigma}_D = \sqrt{2.41^2 + 1.78^2}$$

$$\text{Sigma}_D = 3.0$$

$$\frac{D}{\text{Sigma}_D} = \frac{1.0}{3.0} = .33$$

From Garrett Table 34 at .35

Chances are 64 in 100

Table 18. Comparison of Scores and Significance of the Difference between the Means for the Lower Halves of the Groups on Final Tests

| Lower Half of 1948 Group | | | Lower Half of 1949 Group | | |
|---|---|---|---|---|---|
| Score | d | $d^2$ | Score | d | $d^2$ |
| 67 | 15 | 225 | 63 | 14 | 196 |
| 64 | 12 | 144 | 61 | 12 | 144 |
| 63 | 11 | 121 | 57 | 8 | 64 |
| 62 | 10 | 100 | 56 | 7 | 49 |
| 60 | 8 | 64 | 55 | 6 | 36 |
| 57 | 5 | 25 | 35 | 6 | 36 |
| 55 | 3 | 9 | 54 | 5 | 25 |
| 53 | 1 | 1 | 53 | 4 | 16 |
| 51 | -1 | 1 | 53 | 4 | 16 |
| 50 | -2 | 4 | 53 | 4 | 16 |
| 49 | -3 | 9 | 49 | 0 | 0 |
| 49 | -3 | 9 | 49 | 0 | 0 |
| 48 | -4 | 16 | 48 | -1 | 1 |
| 48 | -4 | 16 | 43 | -6 | 36 |
| 43 | -9 | 81 | 35 | -14 | 196 |
| 41 | -11 | 121 | 35 | -14 | 196 |
| 26 | -26 | 676 | 29 | -20 | 400 |
| 17 \| 8860 | | 17 \| 1622 | 17 \| 8280 | | 17 \| 1427 |
| 52.1 | | 95.4 | 48.7 | | 83.94 |
| Mean = 52 | | Sigma = 9.75 | Mean = 49 | | Sigma = 9.15 |

Sigma$_D$ of Lower Halves

$M_{48} - M_{49} = 52 - 49 = 3.0$

$\text{Sigma}_{M_{48}} = \frac{9.75}{\sqrt{17}} = 2.36$

$\text{Sigma}_{M_{49}} = \frac{9.15}{\sqrt{17}} = 2.22$

$\text{Sigma}_D = \sqrt{2.36^2 + 2.22^2}$

$\text{Sigma}_D = 3.2$

$\frac{D}{\text{Sigma}_D} = \frac{3.0}{3.2} = .93$

From Garrett Table 34 at .95

Chances are 83 in 100

Table 19. Graphs of Achievement on Semi-Final Tests for 1948 and 1949 Groups

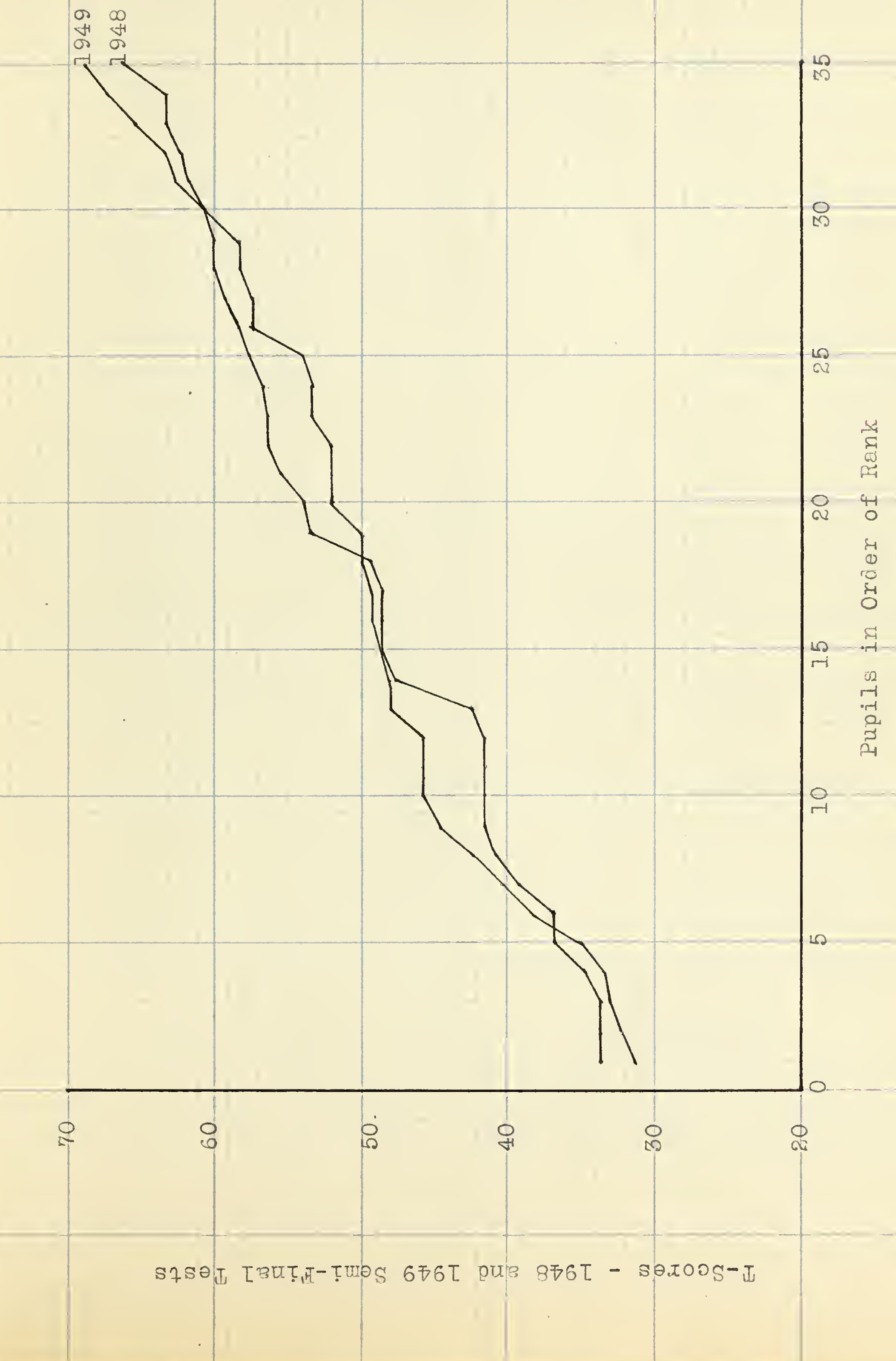
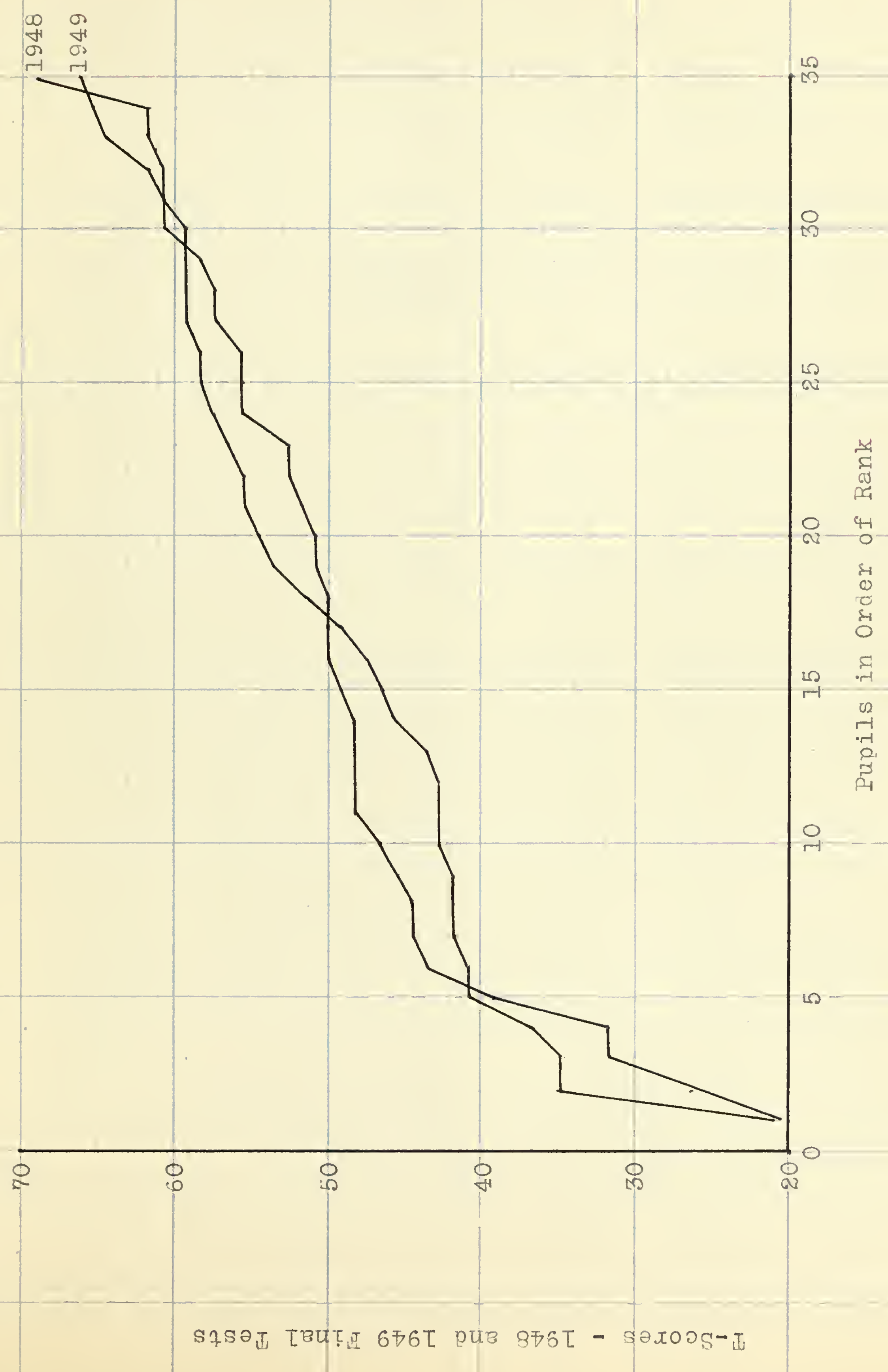


Table 20. Graphs of Achievement on Final Tests for 1948 and 1949 Groups



# APPENDIX B

Table 1. Graphs of Intelligence Quotient versus Frequency of All Eleventh Grade Students and of Plane Geometry Students in 1948

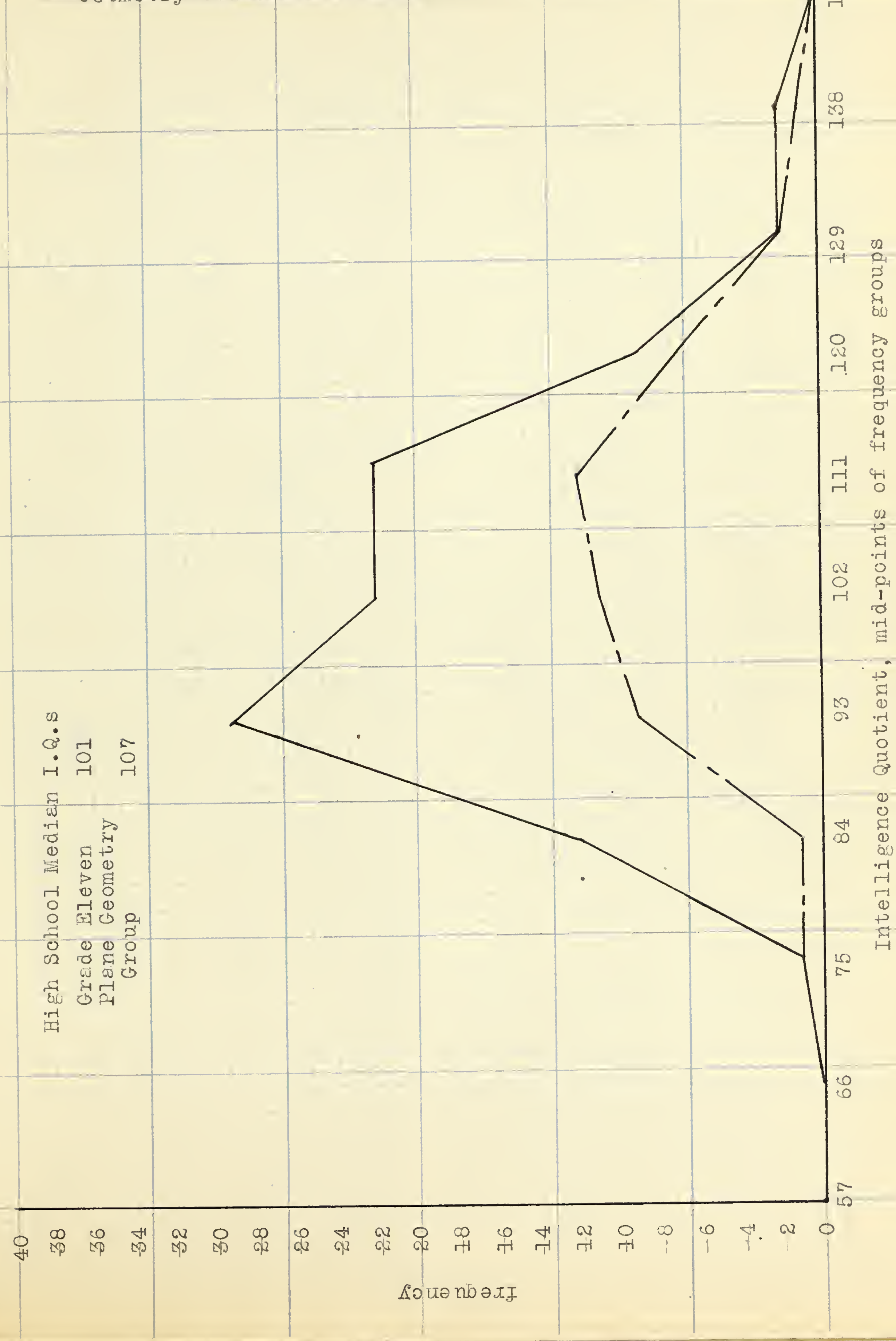


Table 2. Graphs of Intelligence Quotient versus Frequency of All Eleventh-Grade Students and of Plane Geometry Students in 1949

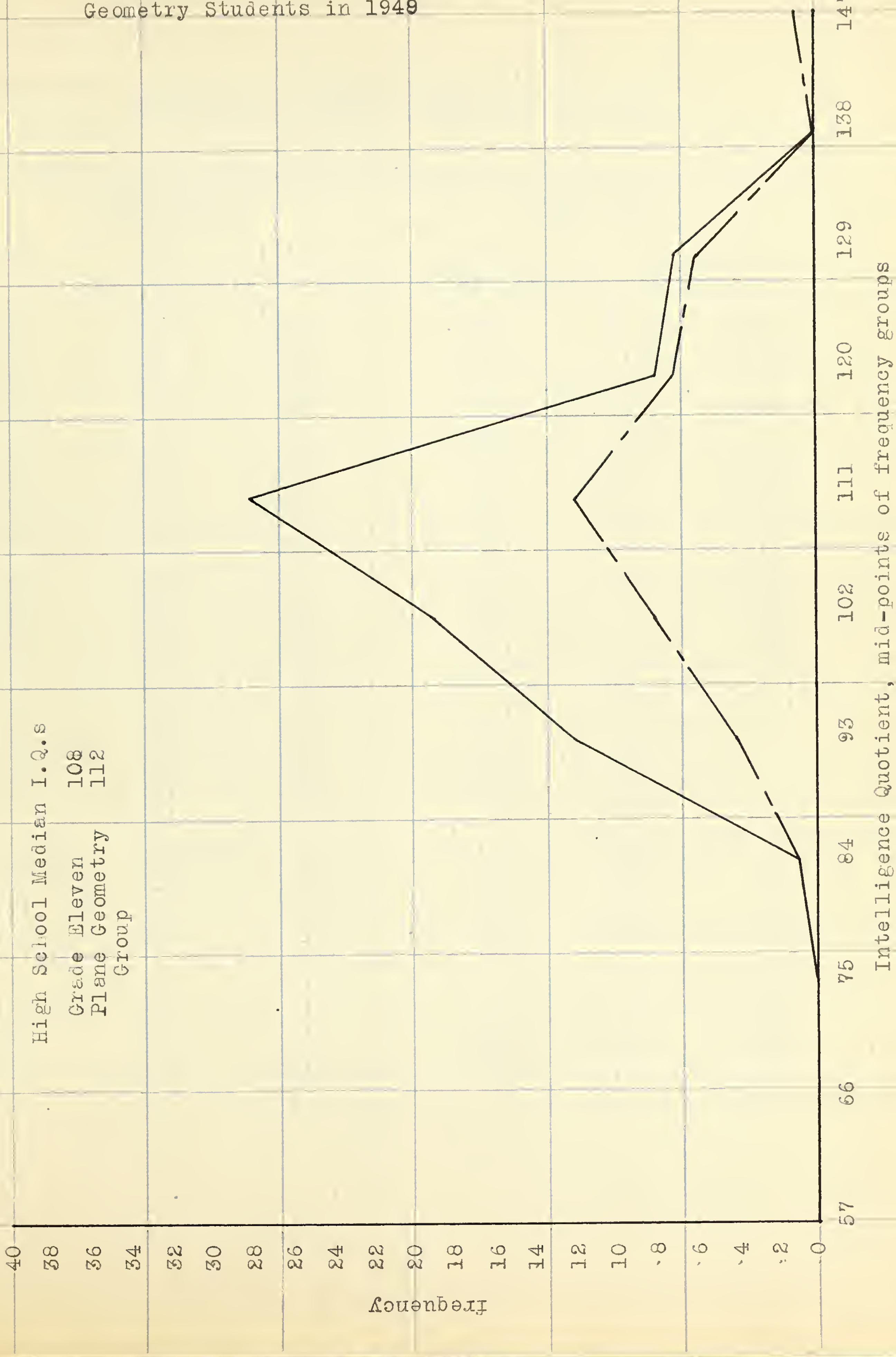


Table 3. Data Used in Calculating Mean Intelligence Quotient and Sigma of Unequated Groups

| 1948 Groups | | | 1949 Groups | | |
|---|---|---|---|---|---|
| IQ | d | $d^2$ | IQ | d | $d^2$ |
| 140 | 35 | 1225 | 143 | 31 | 961 |
| 128 | 23 | 529 | 132 | 20 | 400 |
| 128 | 23 | 529 | 129 | 17 | 289 |
| 124 | 19 | 361 | 126 | 14 | 196 |
| 122 | 17 | 289 | 125 | 13 | 169 |
| 121 | 16 | 256 | 125 | 13 | 169 |
| 120 | 15 | 225 | 125 | 13 | 169 |
| 117 | 12 | 144 | 124 | 12 | 144 |
| 117 | 12 | 144 | 124 | 12 | 144 |
| 116 | 11 | 121 | 123 | 11 | 121 |
| 115 | 10 | 100 | 122 | 10 | 100 |
| 113 | 8 | 64 | 119 | 7 | 49 |
| 112 | 7 | 49 | 118 | 6 | 36 |
| 111 | 6 | 36 | 117 | 5 | 25 |
| 111 | 6 | 36 | 115 | 3 | 9 |
| 110 | 5 | 25 | 113 | 1 | 1 |
| 110 | 5 | 25 | 112 | 0 | |
| 110 | 5 | 25 | 112 | 0 | |
| 109 | 4 | 16 | 112 | 0 | |
| 109 | 4 | 16 | 112 | 0 | |
| 108 | 3 | 9 | 111 | -1 | 1 |
| 107 | 2 | 4 | 110 | -2 | 4 |
| 106 | 1 | 1 | 109 | -3 | 9 |
| 106 | 1 | 1 | 109 | -3 | 9 |
| 104 | -1 | 1 | 108 | -4 | 16 |
| 104 | -1 | 1 | 107 | -5 | 25 |
| 104 | -1 | 1 | 106 | -6 | 36 |
| 102 | -3 | 9 | 106 | -6 | 36 |
| 102 | -3 | 9 | 106 | -6 | 36 |
| 100 | -5 | 25 | 102 | -10 | 100 |

Table 3. (continued)

| 1948 Groups | | | 1949 Groups | | |
|---|---|---|---|---|---|
| IQ | d | $d^2$ | IQ | d | $d^2$ |
| 98 | -7 | 49 | 102 | -10 | 100 |
| 98 | -7 | 49 | 100 | -12 | 144 |
| 98 | -7 | 49 | 98 | -14 | 196 |
| 97 | -8 | 64 | 98 | -14 | 196 |
| 97 | -8 | 64 | 97 | -15 | 225 |
| 97 | -8 | 64 | 97 | -15 | 225 |
| 97 | -8 | 64 | 95 | -17 | 289 |
| 96 | -9 | 81 | 95 | -17 | 289 |
| 95 | -10 | 100 | 86 | -26 | 576 |
| 94 | -11 | 121 | | | |
| 92 | -13 | 169 | | | |
| 90 | -15 | 225 | | | |
| 88 | -17 | 289 | | | |
| 78 | -27 | 729 | | | |
| 4601 | | 6393 | 4370 | | 5594 |

**1948 Groups**

Mean = $44\overline{)4601.00}$ = 104.56

Mean = 105

Sigma = $\sqrt{\dfrac{6393.00}{44}} = \sqrt{145.29}$ = 12.05

Sigma = 12.1

**1949 Groups**

Mean = $39\overline{)4370.00}$ = 112.05

Mean = 112

Sigma = $\sqrt{\dfrac{5594.00}{44}} = \sqrt{143.43}$ = 11.98

Sigma = 12.0

Table 4. Data Used in Calculating Mean Intelligence Quotient and Sigma of Equated Groups - First Trial

| 1948 Groups | | | 1949 Groups | | |
|---|---|---|---|---|---|
| IQ | d | $d^2$ | IQ | d | $d^2$ |
| 140 | 34 | 1156 | 129 | 15 | 225 |
| 128 | 22 | 484 | 126 | 12 | 144 |
| 128 | 22 | 484 | 125 | 11 | 121 |
| 124 | 18 | 324 | 125 | 11 | 121 |
| 122 | 16 | 256 | 125 | 11 | 121 |
| 121 | 15 | 225 | 124 | 10 | 100 |
| 120 | 14 | 196 | 124 | 10 | 100 |
| 117 | 11 | 121 | 123 | 9 | 81 |
| 117 | 11 | 121 | 122 | 8 | 64 |
| 116 | 10 | 100 | 119 | 5 | 25 |
| 115 | 9 | 81 | 118 | 4 | 16 |
| 113 | 7 | 49 | 117 | 3 | 9 |
| 112 | 6 | 36 | 115 | 1 | 1 |
| 111 | 5 | 25 | 113 | -1 | 1 |
| 111 | 5 | 25 | 112 | -2 | 4 |
| 110 | 4 | 16 | 112 | -2 | 4 |
| 110 | 4 | 16 | 112 | -2 | 4 |
| 110 | 4 | 16 | 112 | -2 | 4 |
| 109 | 3 | 9 | 111 | -3 | 9 |
| 109 | 3 | 9 | 110 | -4 | 16 |
| 108 | 2 | 4 | 109 | -5 | 25 |
| 107 | 1 | 1 | 109 | -5 | 25 |
| 106 | 0 | | 108 | -6 | 36 |
| 106 | 0 | | 107 | -7 | 49 |
| 104 | -2 | 4 | 106 | -8 | 64 |
| 104 | -2 | 4 | 106 | -8 | 64 |
| 104 | -2 | 4 | 106 | -8 | 64 |
| 102 | -4 | 16 | 102 | -12 | 144 |
| 102 | -4 | 16 | 102 | -12 | 144 |
| 100 | -6 | 36 | 100 | -14 | 196 |

Table 4. (continued)

| 1948 Groups | | | 1949 Groups | | |
|---|---|---|---|---|---|
| IQ | d | $d^2$ | IQ | d | $d^2$ |
| 98 | -8 | 64 | 98 | -16 | 256 |
| 98 | -8 | 64 | 98 | -16 | 256 |
| 98 | -8 | 64 | 97 | -17 | 289 |
| 97 | -9 | 81 | 97 | -17 | 289 |
| 97 | -9 | 81 | 95 | -19 | 361 |
| 97 | -9 | 81 | 95 | -19 | 361 |
| 97 | -9 | 81 | 86 | -28 | 784 |
| 96 | -10 | 100 | | | |
| 95 | -11 | 121 | | | |
| 94 | -12 | 144 | | | |
| 4253 | | 4715 | 4207 | | 4577 |

Mean = 40 $\overline{)4253.00}$ = 106.32

Mean = 106

Sigma = $\sqrt{117.87}$ = 10.86 (40 $\overline{)4715.00}$)

Sigma = 10.9

Mean = 37 $\overline{)4207.00}$ = 113.64

Mean = 114

Sigma = $\sqrt{123.7}$ = 11.14 (37 $\overline{)4577.00}$)

Sigma = 11.1

Table 5. Data Used in Calculating Mean Intelligence Quotient and Sigma of Equated Groups

| 1948 Groups | | | 1949 Groups | | |
|---|---|---|---|---|---|
| IQ | d | $d^2$ | IQ | d | $d^2$ |
| 140 | 32 | 1024 | 125 | 15 | 225 |
| 128 | 20 | 400 | 125 | 15 | 225 |
| 128 | 20 | 400 | 125 | 15 | 225 |
| 124 | 16 | 256 | 124 | 14 | 196 |
| 122 | 14 | 196 | 124 | 14 | 196 |
| 121 | 13 | 169 | 123 | 13 | 169 |
| 120 | 12 | 144 | 122 | 12 | 144 |
| 117 | 9 | 81 | 119 | 9 | 81 |
| 117 | 9 | 81 | 118 | 8 | 64 |
| 116 | 8 | 64 | 117 | 7 | 49 |
| 115 | 7 | 49 | 115 | 5 | 25 |
| 113 | 5 | 25 | 113 | 3 | 9 |
| 112 | 4 | 16 | 112 | 2 | 4 |
| 111 | 3 | 9 | 112 | 2 | 4 |
| 111 | 3 | 9 | 112 | 2 | 4 |
| 110 | 2 | 4 | 112 | 2 | 4 |
| 110 | 2 | 4 | 111 | 1 | 1 |
| 110 | 2 | 4 | 110 | 0 | |
| 109 | 1 | 1 | 109 | -1 | 1 |
| 109 | 1 | 1 | 109 | -1 | 1 |
| 108 | 0 | | 108 | -2 | 4 |
| 107 | -1 | 1 | 107 | -3 | 9 |
| 106 | -2 | 4 | 106 | -4 | 16 |
| 106 | -2 | 4 | 106 | -4 | 16 |
| 104 | -4 | 16 | 106 | -4 | 16 |
| 104 | -4 | 16 | 102 | -8 | 64 |
| 104 | -4 | 16 | 102 | -8 | 64 |
| 102 | -6 | 36 | 100 | -10 | 100 |
| 102 | -6 | 36 | 98 | -12 | 144 |
| 100 | -8 | 64 | 98 | -12 | 144 |

Table 5. (continued)

| 1948 Groups | | | 1949 Groups | | |
|---|---|---|---|---|---|
| IQ | d | $d^2$ | IQ | d | $d^2$ |
| 98 | -10 | 100 | 97 | -13 | 169 |
| 98 | -10 | 100 | 97 | -13 | 169 |
| 98 | -10 | 100 | 95 | -15 | 225 |
| 97 | -11 | 121 | 95 | -15 | 225 |
| 97 | -11 | 121 | 86 | -24 | 576 |
| 3774 | | 3672 | 3840 | | 3568 |

Mean = 35 $\overline{)3774.0}$ = 107.8

Mean = 108

Sigma = $\sqrt{\frac{3672.0}{35}} = \sqrt{104.9}$ = 10.24

Sigma = 10.2

Mean = 35 $\overline{)3840.0}$ = 109.7

Mean = 110

Sigma = $\sqrt{\frac{3568.0}{35}} = \sqrt{101.9}$ = 10.09

Sigma = 10.1

Table 6. Intelligence Quotients and Previous Marks in Mathematics of Pupils in 1948 Group

| Pupil | IQ | Previous Marks in Mathematics Grades 7 | 8 | 9 | 10 | Av. |
|---|---|---|---|---|---|---|
| 1 | 140 | A | A | B | C+ | B+ |
| 2 | 128 | B+ | A | B+ | B+ | B+ |
| 3 | 128 | B+ | B | B | B | B |
| 4 | 124 | na* | C | C+ | C | C |
| 5 | 122 | C+ | B | B | C+ | C+ |
| 6 | 121 | B | A | B | C+ | B |
| 7 | 120 | B+ | A | B | C+ | B |
| 8 | 117 | D+ | C+ | D+ | C | C |
| 9 | 117 | na | na | A | B+ | A |
| 10 | 116 | B | B | A | A | B+ |
| 11 | 115 | B | B | C+ | C+ | C+ |
| 12 | 113 | C | B | B | B | C+ |
| 13 | 112 | B | B+ | B+ | A | B+ |
| 14 | 111 | B | C | C | C | C |
| 15 | 111 | na | na | B+ | B | B |
| 16 | 110 | A | A | B+ | B | B+ |
| 17 | 110 | C | C | C | C | C |
| 18 | 110 | C | C | D | C | C |
| 19 | 109 | C | C+ | C+ | C | C+ |
| 20 | 109 | D | C | C | C | C |
| 21 | 108 | B- | C | C | C | C |
| 22 | 107 | C | C | C | D | C |
| 23 | 106 | B | B | B | C+ | B |
| 24 | 106 | C+ | C+ | B | B | C+ |
| 25 | 104 | D | C | C+ | D | D+ |
| 26 | 104 | na | na | B | B | B |
| 27 | 104 | B | B- | C | D | C |
| 28 | 102 | na | na | C | B | C+ |
| 29 | 102 | B | B | C+ | D | C+ |
| 30 | 100 | D+ | C | C | D | D+ |

Table 6 (continued)

Previous Marks in Mathematics

| | | Grades | | | | |
|---|---|---|---|---|---|---|
| Pupil | IQ | 7 | 8 | 9 | 10 | Av. |
| 31 | 98 | D | C | C | D | D+ |
| 32 | 98 | D | C+ | C | C | C |
| 33 | 98 | D+ | C | D+ | D | D+ |
| 34 | 97 | D | D | D+ | D | D |
| 35 | 97 | na | na | D | D+ | D |
| | | | | | Mean = | 2.43 |
| | | | | | Conv. Mean = | C+ |

*na indicates not available

In average letter grades, following conversion scale was used:

A = 4.0; B+ = 3.5; B = 3.0; C+ = 2.5;
C = 2.0; D+ = 1.5; D = 1.0

Table 7. Intelligence Quotients and Previous Marks in Mathematics of Pupils in 1949 Group

| | | Previous Marks in Mathematics | | | | |
|---|---|---|---|---|---|---|
| | | Grades | | | | |
| Pupil | IQ | 7 | 8 | 9 | 10 | Av. |
| 1 | 125 | B+ | A | A | A | A |
| 2 | 125 | na | B+ | A | B | B+ |
| 3 | 125 | na | B | D | C | C |
| 4 | 124 | A | A | A | B+ | A |
| 5 | 124 | B | B+ | B+ | B | B |
| 6 | 123 | A | C+ | B+ | B | B |
| 7 | 122 | B | C+ | C+ | C | C+ |
| 8 | 119 | na | na | C+ | C+ | C+ |
| 9 | 118 | C+ | C+ | C | C | C |
| 10 | 117 | C+ | C+ | C+ | C+ | C+ |
| 11 | 115 | B | C | B | C | C+ |
| 12 | 113 | C | C | B | B | C+ |
| 13 | 112 | B+ | B | B | C | B |
| 14 | 112 | B | C | C+ | C+ | C+ |
| 15 | 112 | A | B | C+ | C+ | B |
| 16 | 112 | B | B+ | B+ | B | B |
| 17 | 111 | C | D+ | C+ | C+ | C |
| 18 | 110 | C | C+ | C | C+ | C |
| 19 | 109 | B+ | A | B | B | B+ |
| 20 | 109 | A | C+ | C+ | B | B |
| 21 | 108 | B | B | C+ | C | C+ |
| 22 | 107 | D | C | B | C+ | C |
| 23 | 106 | B | B | A | B | B |
| 24 | 106 | D | D | D | D | D |
| 25 | 106 | C | C+ | D+ | E | D+ |
| 26 | 102 | na | na | D+ | E | D |
| 27 | 102 | C+ | C+ | C+ | D | C |
| 28 | 100 | B | C+ | C | D+ | C |
| 29 | 98 | na | B | C | C | C+ |
| 30 | 98 | C+ | C | D+ | C | C |

Table 7 (continued)

Previous Marks in Mathematics

| | | Grades | | | | |
|---|---|---|---|---|---|---|
| Pupil | IQ | 7 | 8 | 9 | 10 | Av. |
| 31 | 97 | A | B | A | A | B+ |
| 32 | 97 | C | C+ | D | D+ | D+ |
| 33 | 95 | na | na | D+ | D | D |
| 34 | 95 | B | C | C | B | C+ |
| 35 | 86 | C- | C | C+ | C+ | C+ |
| | | | | | Mean | = 2.471 |
| | | | | | Conv. Mean | = C+ |

na indicates not available

In averaging letter grades, following conversion scale was used:

A = 4.0; B+ = 3.5; B = 3.0; C+ = 2.5;
C = 2.0; D+ = 1.5; D = 1.0

Table 8. Data from Boston University School and College Relations Cooperative Testing Service Vocational Guidance Battery for Pupils in 1948 Group

| Pupil | Intel. Quot. | Chron. Age | Problem Solving | Reading Comp. | Spatial Relations |
|---|---|---|---|---|---|
| 1 | 140 | 16-7 | 11 | 224 | 61 |
| 2 | 128 | 15-10 | 11 | 184 | 44 |
| 3 | 128 | 16-5 | 9 | 171 | 56 |
| 4 | 124 | 16-10 | 9 | 169 | 51 |
| 5 | 122 | 16-5 | 9 | 164 | 58 |
| 6 | 121 | 16-8 | 10 | 186 | 56 |
| 7 | 120 | 15-6 | 5 | 183 | 45 |
| 8 | 117 | 16-1 | 7 | 177 | 42 |
| 9 | 117 | 16-6 | 9 | 173 | 54 |
| 10 | 116 | 15-8 | 7 | 180 | 49 |
| 11 | 115 | 16-6 | 6 | 161 | 38 |
| 12 | 113 | 16-7 | 9 | 200 | 35 |
| 13 | 112 | 16-3 | 4 | 182 | 55 |
| 14 | 111 | 16-4 | 7 | 188 | 44 |
| 15 | 111 | 15-11 | 4 | 160 | 49 |
| 16 | 110 | 16-7 | 9 | 169 | 51 |
| 17 | 110 | 16-3 | 8 | 182 | 46 |
| 18 | 110 | 15-9 | 7 | 146 | 32 |
| 19 | 109 | 16-7 | 6 | 121 | 36 |
| 20 | 109 | 15-11 | 5 | 167 | 47 |
| 21 | 108 | 15-11 | 5 | 181 | 28 |
| 22 | 107 | 16-2 | 7 | 170 | 44 |
| 23 | 106 | 15-11 | 5 | 199 | 53 |
| 24 | 106 | 15-11 | 8 | 163 | 60 |
| 25 | 104 | 15-11 | 8 | 125 | 57 |
| 26 | 104 | 15-8 | 5 | 157 | 40 |
| 27 | 104 | 16-2 | 11 | 126 | 46 |
| 28 | 102 | 16-6 | 5 | 148 | 28 |
| 29 | 102 | 16-1 | 6 | 141 | 37 |
| 30 | 100 | 15-2 | 8 | 143 | 50 |

Table 8 (continued)

| Pupil | Intel. Quot. | Chron. Age | Problem Solving | Reading Comp. | Spatial Relations |
|---|---|---|---|---|---|
| 31 | 98 | 16-8 | 7 | 135 | 43 |
| 32 | 98 | 16-6 | 5 | 149 | 47 |
| 33 | 98 | 15-10 | 5 | 156 | 34 |
| 34 | 97 | 16-7 | 4 | 120 | 46 |
| 35 | 97 | 16-3 | 5 | 167 | 43 |
| Range | 97-140 | (15-2)(16-10) | 4-11 | 120-224 | 28-61 |
| Median | 110 | 16-3 | 7 | 167 | 46 |
| Mean | 108 | 16-2 | 7 | 165 | 46 |
| Sigma | 10.2 | | | | |

Table 9. Data from Boston University School and College Relations Cooperative Testing Service Vocational Guidance Battery for Pupils in 1949 Group

| Pupil | Intel. Quot. | Chron. Age | Problem Solving | Reading Comp. | Spatial Relations |
|---|---|---|---|---|---|
| 1 | 125 | 16-1 | 10 | 169 | 46 |
| 2 | 125 | 16-7 | 10 | 161 | 50 |
| 3 | 125 | 15-8 | 8 | 195 | 47 |
| 4 | 124 | 16-0 | 12 | 219 | 37 |
| 5 | 124 | 15-10 | 7 | 193 | 45 |
| 6 | 123 | 17-2 | 4 | 200 | 36 |
| 7 | 122 | 15-10 | 7 | 163 | 47 |
| 8 | 119 | 16-0 | 8 | 171 | 40 |
| 9 | 118 | 16-8 | 7 | 161 | 41 |
| 10 | 117 | 15-5 | 5 | 144 | 44 |
| 11 | 115 | 16-5 | 8 | 190 | 39 |
| 12 | 113 | 15-10 | 5 | 178 | 44 |
| 13 | 112 | 15-11 | 6 | 153 | 39 |
| 14 | 112 | 16-5 | 5 | 164 | 48 |
| 15 | 112 | 16-1 | 4 | 170 | 33 |
| 16 | 112 | 16-1 | 7 | 162 | 55 |
| 17 | 111 | 16-5 | 6 | 168 | 30 |
| 18 | 110 | 16-1 | 8 | 168 | 46 |
| 19 | 109 | 15-10 | 7 | 193 | 54 |
| 20 | 109 | 17-6 | 8 | 157 | 35 |
| 21 | 108 | 16-2 | 6 | 232 | 36 |
| 22 | 107 | 15-7 | 8 | 140 | 44 |
| 23 | 106 | 15-10 | 7 | 162 | 39 |
| 24 | 106 | 16-5 | 4 | 125 | 26 |
| 25 | 106 | 17-0 | 4 | 105 | 35 |
| 26 | 102 | 16-4 | 7 | 142 | 39 |
| 27 | 102 | 15-7 | 5 | 178 | 42 |
| 28 | 100 | 15-11 | 8 | 172 | 33 |
| 29 | 98 | 17-0 | 8 | 135 | 43 |
| 30 | 98 | 16-1 | 3 | 130 | 25 |

Table 9 (continued)

| Pupil | Intel. Quot. | Chron. Age | Problem Solving | Reading Comp. | Spatial Relations |
|---|---|---|---|---|---|
| 31 | 97 | 16-6 | 6 | 162 | 42 |
| 32 | 97 | 16-7 | 4 | 135 | 33 |
| 33 | 95 | 18-4 | 5 | 97 | 27 |
| 34 | 95 | 15-9 | 6 | 141 | 41 |
| 35 | 86 | 15-10 | 4 | 145 | 33 |
| Range | 86-125 | (15-5)(18-4) | 3-10 | 97-232 | 25-55 |
| Median | 110 | 16-1 | 7 | 162 | 40 |
| Mean | 110 | 16-0 | 7 | 165 | 40 |
| Sigma | 10.1 | | | | |

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Date Due

| | | | |
|---|---|---|---|
| APR 26 1958 | | Rm Use | |
| OCT 9 1958 | | | |
| OCT 23 1958 | | MAR 3 | 1962 |
| | | MAR 17 1962 | |
| | | JUL 24 1962 | |
| NOV 10 1958 | | DEC 15 1962 | |
| DEC 10 1958 | | FEB 26 1963 | |
| Rm Use | | | |
| JUL 28 1960 | | | |
| AUG 4 1960 | | JAN 17 | |
| MAR 25 1961 | | | |
| APR 12 1961 | | | |
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| MAY 13 1961 | | | |

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Jacobs, J.W.
1949

Jacobs, J.W.
An experiment in the teaching of plane geometry....

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MAR 3 1962
002 573
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409 559
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Jacobs, J.W.
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